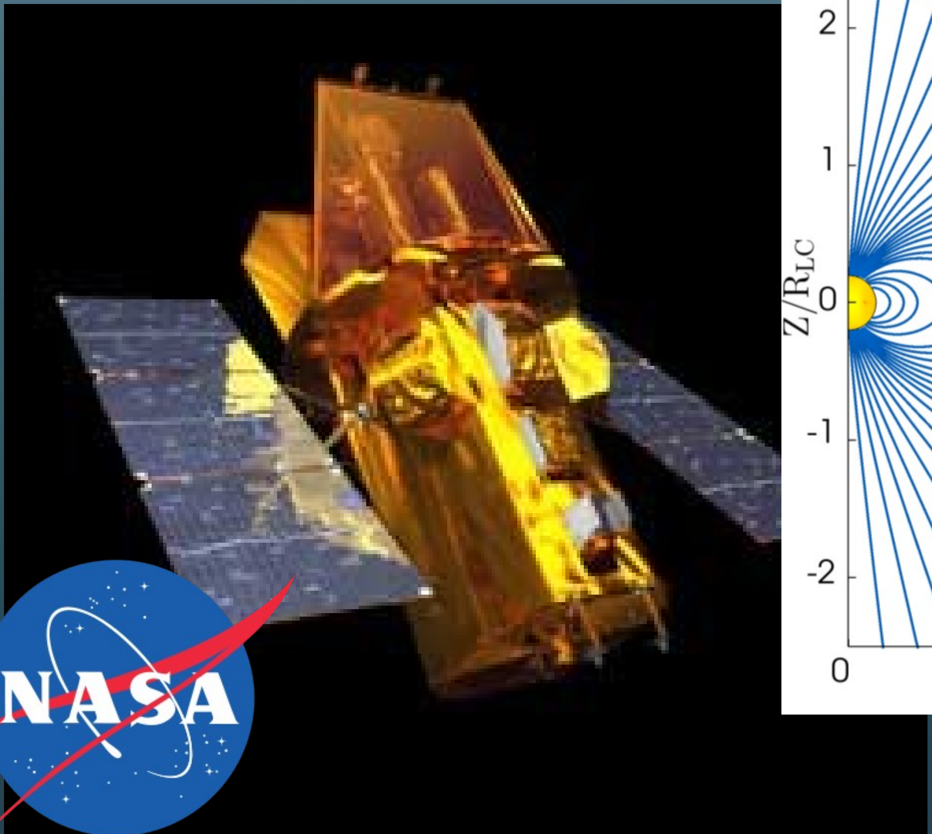
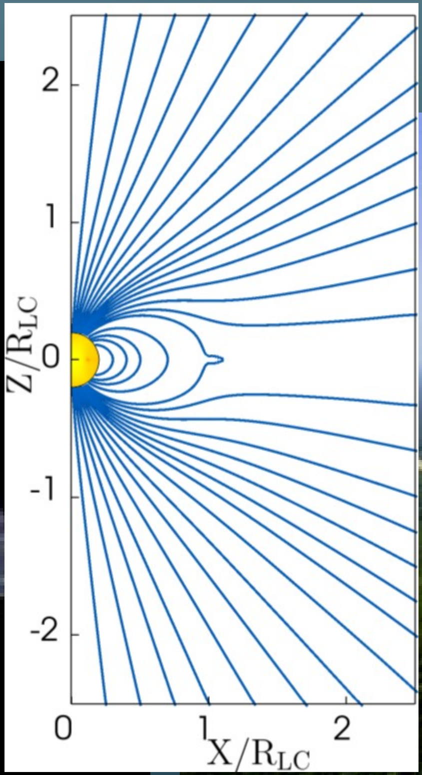
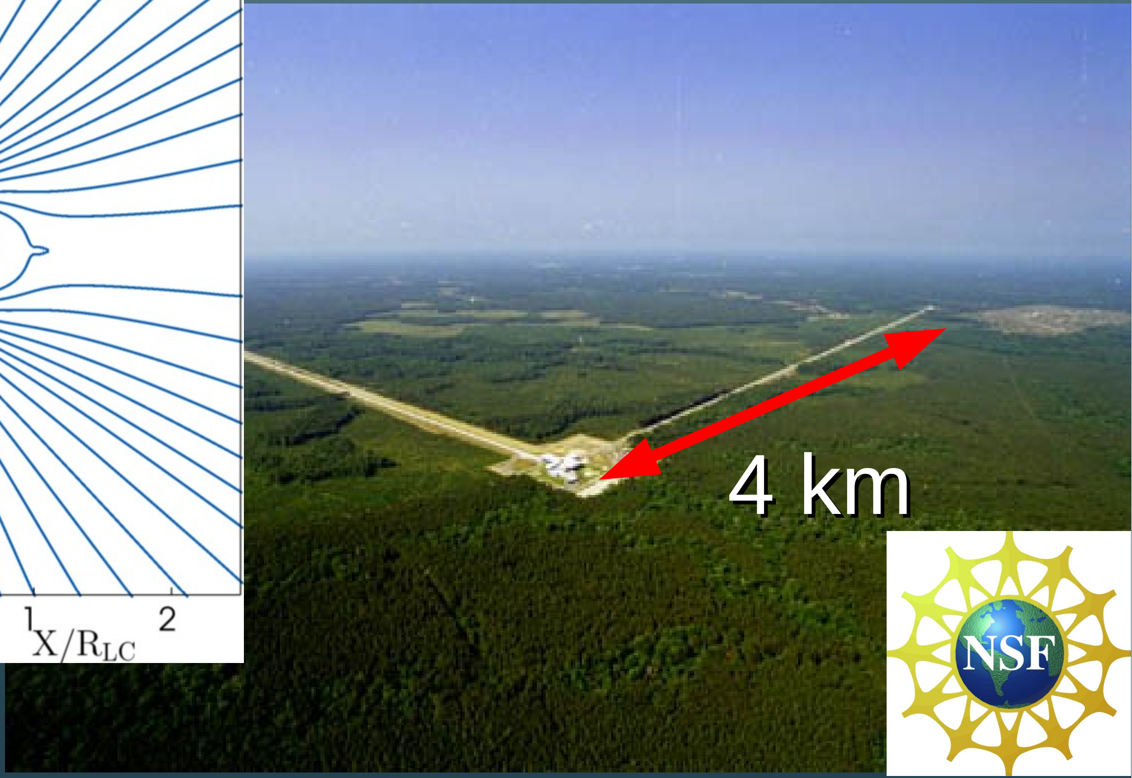
Electromagnetic Counterparts to Gravitational Wave Detections: Bridging the Gap between Theory and Observation

Prof. Zach Etienne, West Virginia University







General Relativity, *Briefly*

- Special Relativity: Speed of light *c* is
  - the same, *no matter how fast you move*

General Relativity, *Briefly*

- Special Relativity: Speed of light *c* is
  - the same, *no matter how fast you move*

Consequences

→ moving clocks tick more slowly
→ moving objects' length become squished
→ relativistic effects greatest if moving near *c*

General Relativity, *Briefly*

- Special Relativity: Speed of light *c* is
  - the same, *no matter how fast you move*

Consequences

→ moving clocks tick more slowly
→ moving objects' length become squished
→ relativistic effects greatest if moving near *c*

General Relativity:

- *Einstein's great insight: The opposite holds true*!

General Relativity, *Briefly*

- Special Relativity: Speed of light *c* is
  - the same, *no matter how fast you move*

Consequences

→ moving clocks tick more slowly
→ moving objects' length become squished
→ relativistic effects greatest if moving near *c*

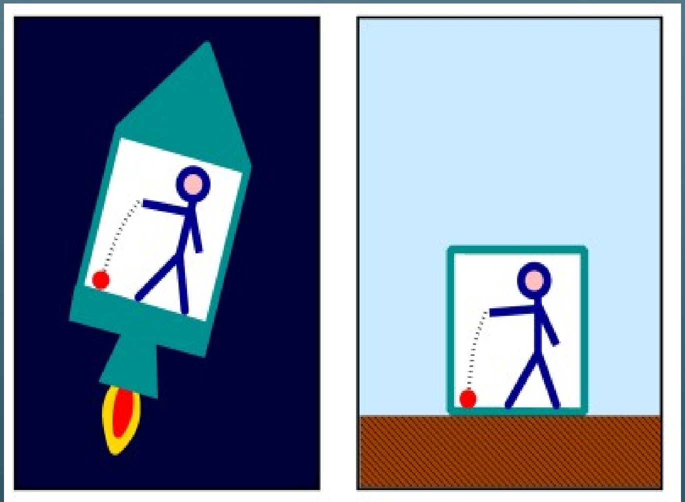
General Relativity:

- *Einstein's great insight: The opposite holds true*!

- **Gravity:** Massive objects
  - Slow down time & squish space around them, leading to <u>permanent acceleration field</u>

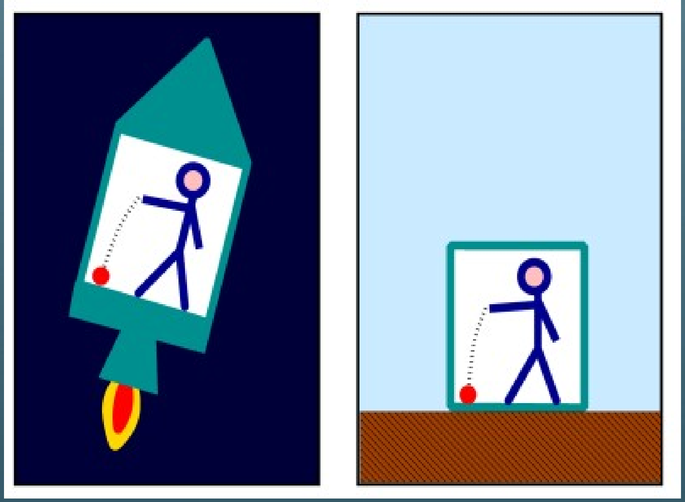
Theory of General Relativity

- Assumes *special* relativity is true, *in addition*:
  - A gravitational acceleration is the same as a normal acceleration: “Equivalence Principle”

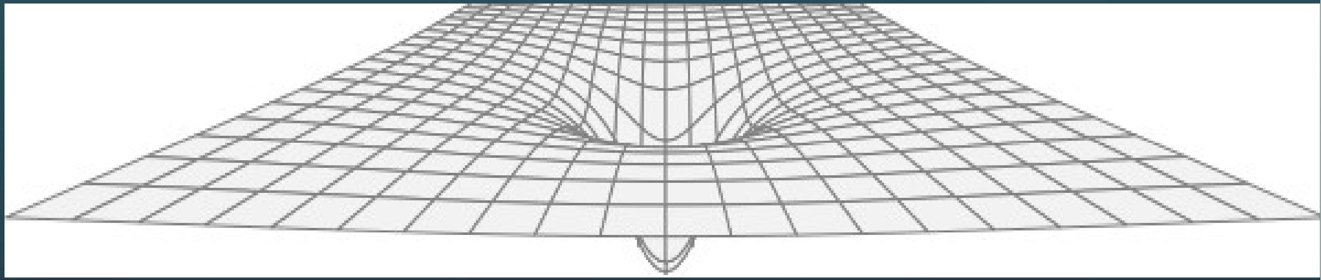


Theory of General Relativity

- Assumes *special* relativity is true, *in addition*:
  - A gravitational acceleration is the same as a normal acceleration: "Equivalence Principle"



- Consequence:
  - Gravity = "permanent acceleration around anything with mass"
    - *Curving* space, and
    - *Slowing down* time



Theory of General Relativity: Summary

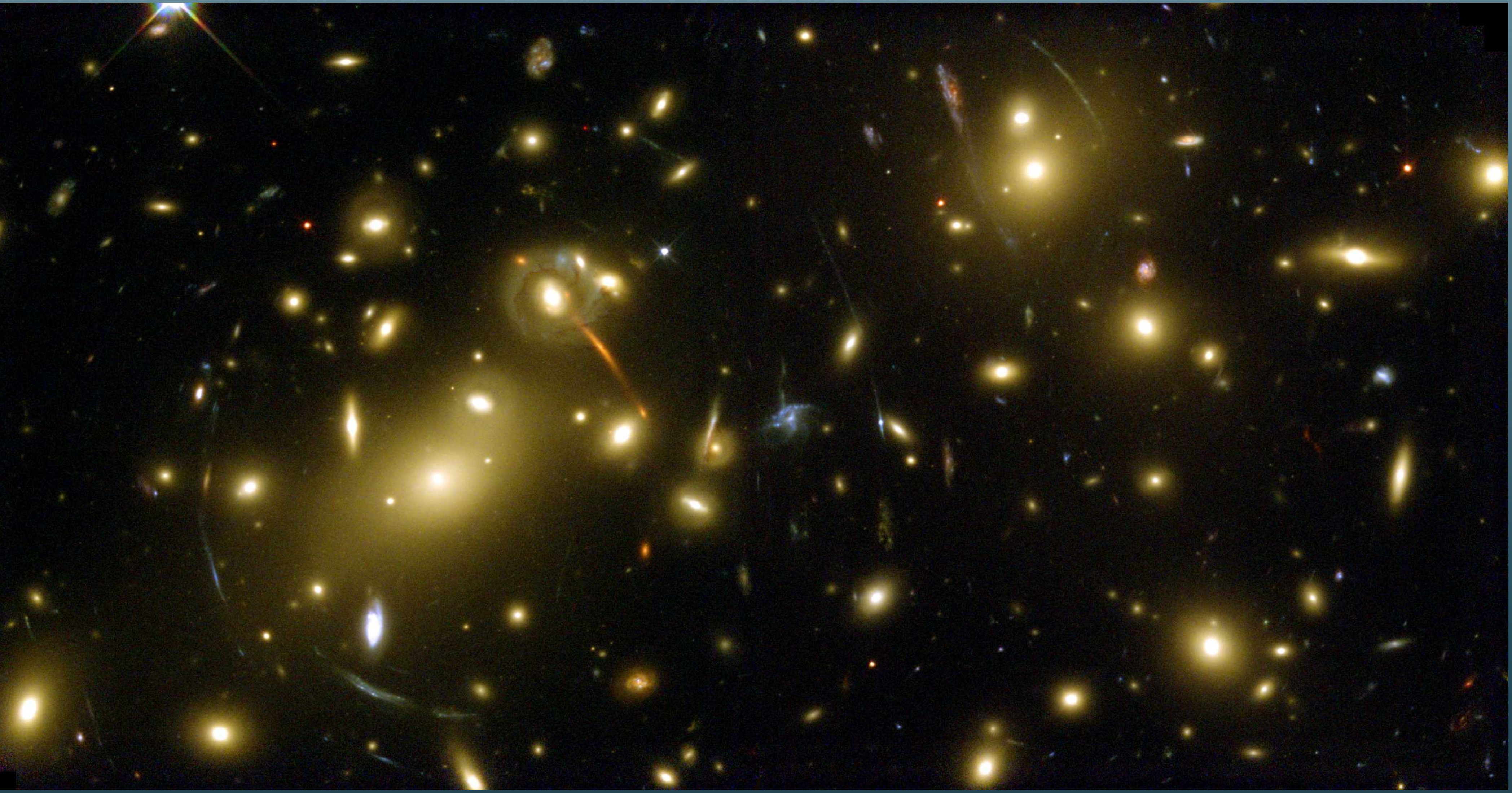
- Gravity curves space & slows down time
  - Clocks on ground tick more slowly than those in hot air balloon
  - Meter sticks standing on ground are shorter than those in hot air balloon
  - Path of light *bends* when traveling around massive object

Predictions of General Relativity: Gravitational Lensing

Predictions of General Relativity: Gravitational Lensing

Predictions of General Relativity: Gravitational Lensing

Predictions of General Relativity: Gravitational Lensing

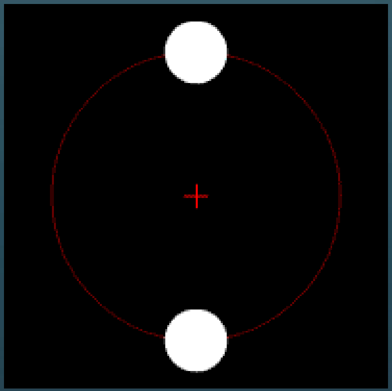


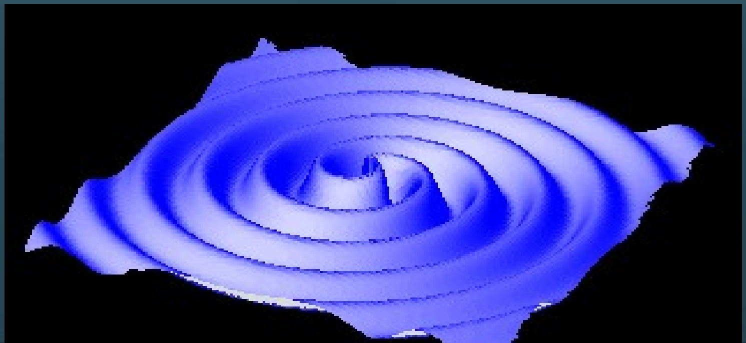
Predictions of General Relativity: Gravitational Waves

- **Newtonian Gravity:**
  - Information about changing gravitational fields propagates *infinitely* fast
- **General Relativity:**
  - Information about changing gravitational fields propagates at *c*, results in *gravitational waves*

Predictions of General Relativity: Gravitational Waves

- **Newtonian Gravity:**
  - Information about changing gravitational fields propagates *infinitely* fast
- **General Relativity:**
  - Information about changing gravitational fields propagates at *c*, results in *gravitational waves*
  - Binary system:
    - Gravitational waves carry away orbital energy & angular momentum

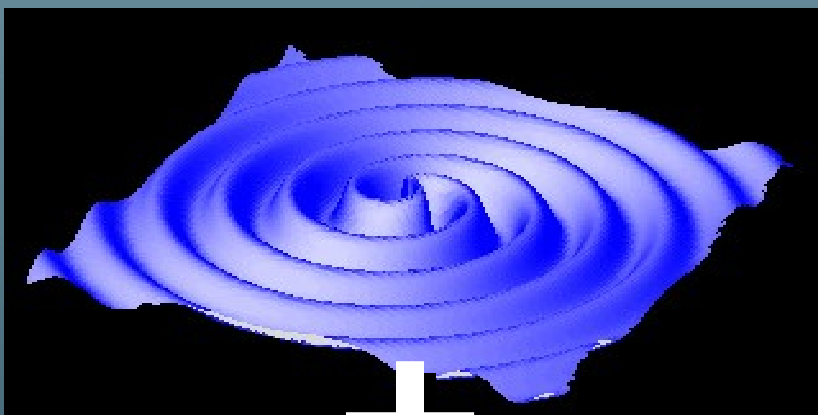




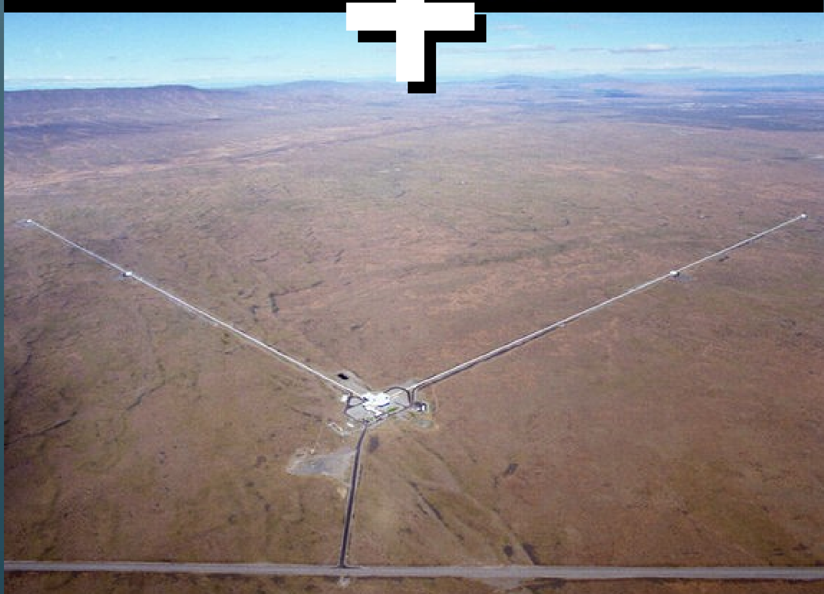
Predictions of General Relativity: Gravitational Waves

- *Strongest waves from objects orbiting near c*
  - Black holes & neutron stars; Sun-like → destroyed
- These waves *detectable*, but *extremely* weak!
  - **LIGO**: About 1/1000 width of proton

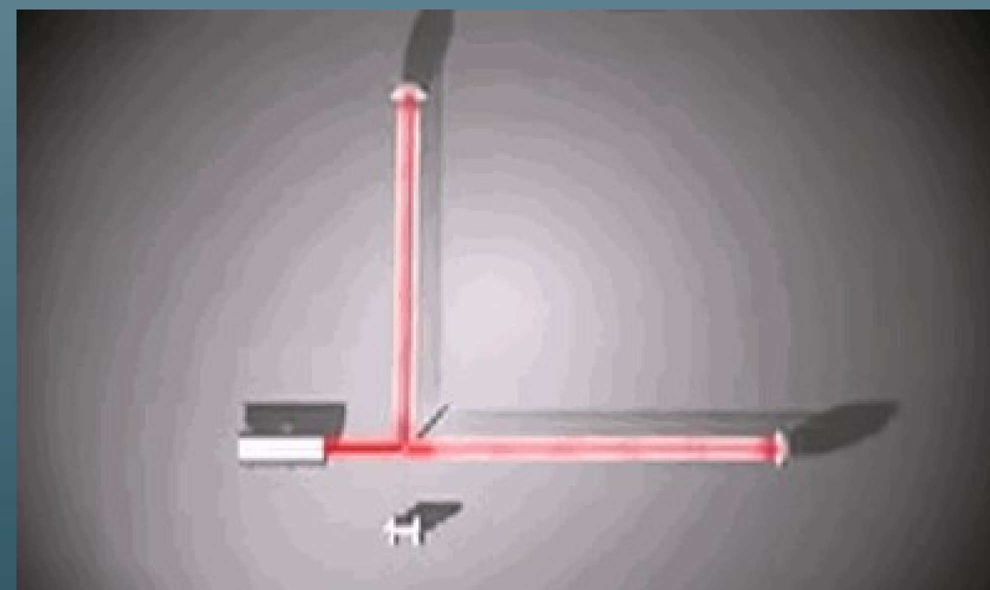
What happens when gravitational waves pass through the detector?

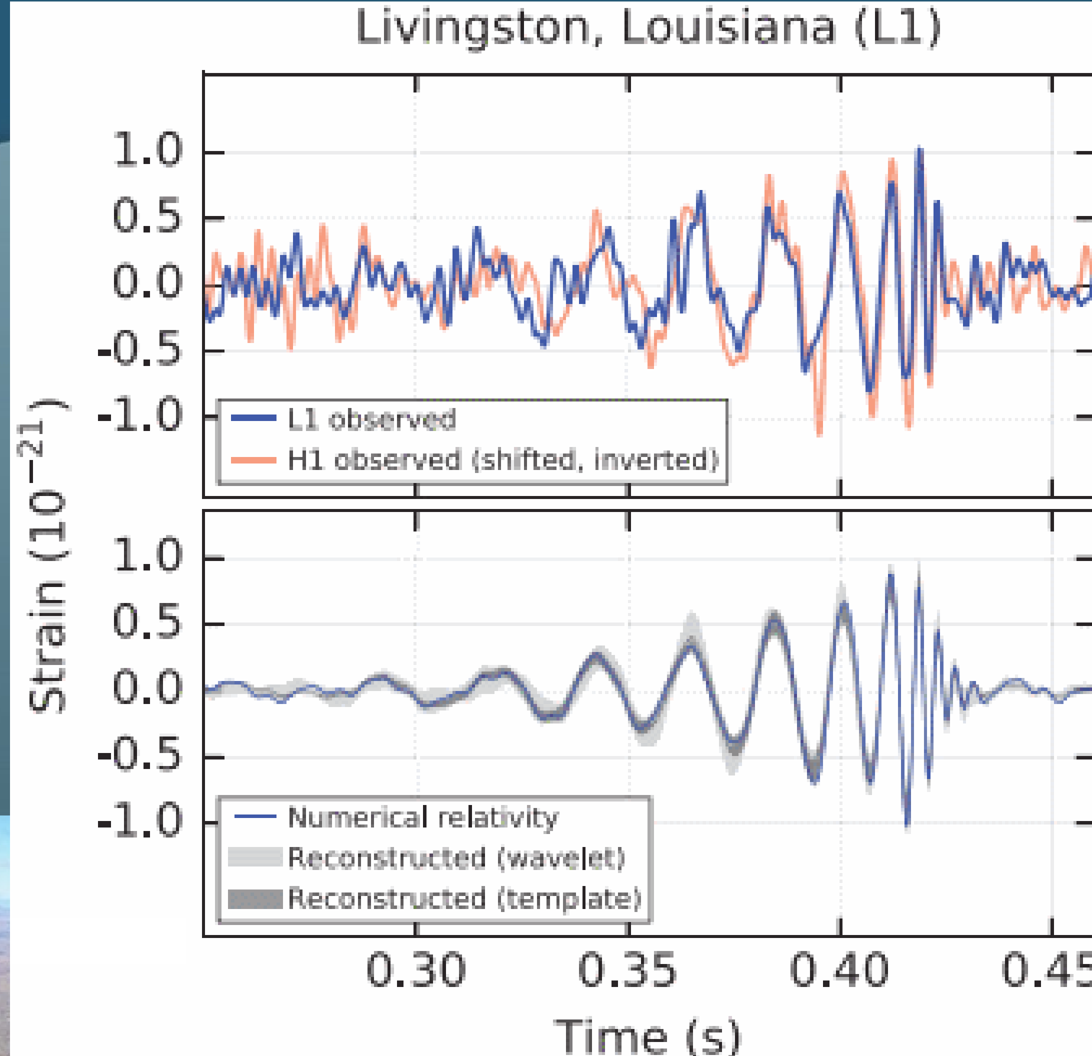


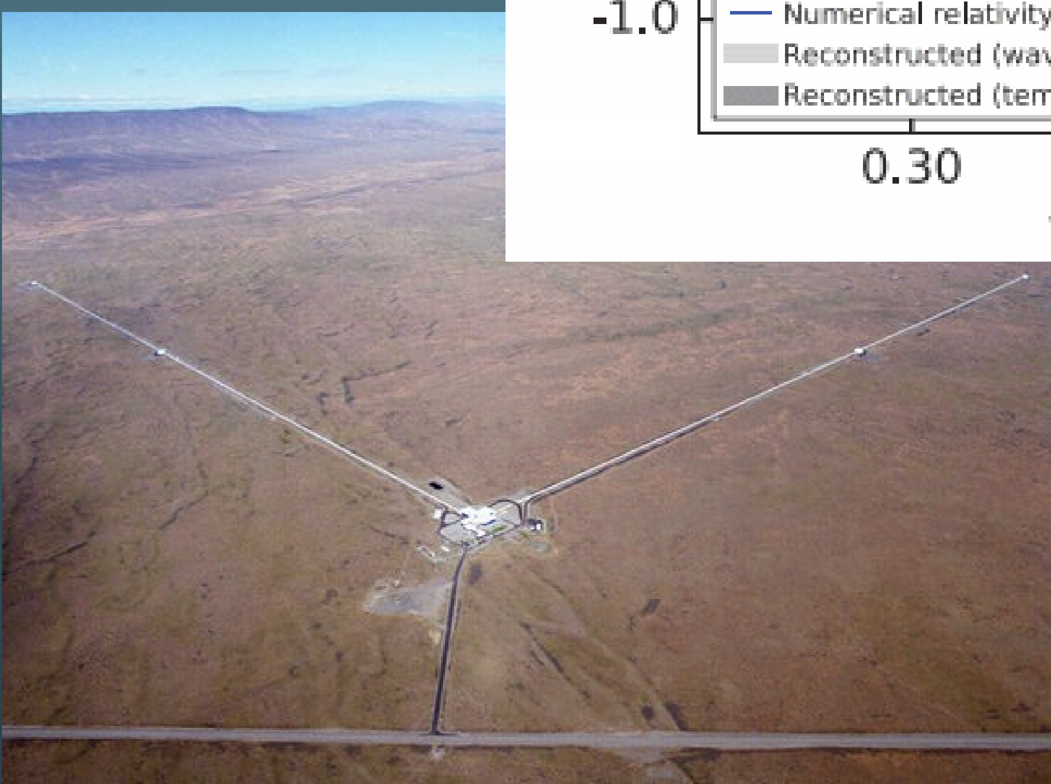
+

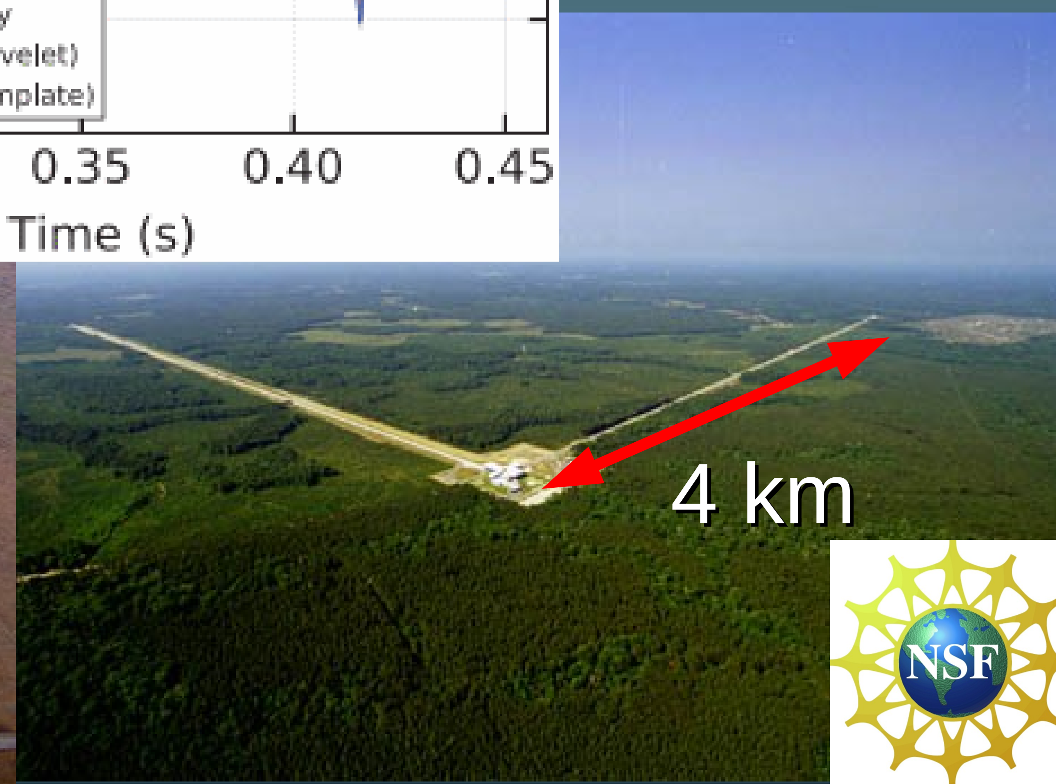


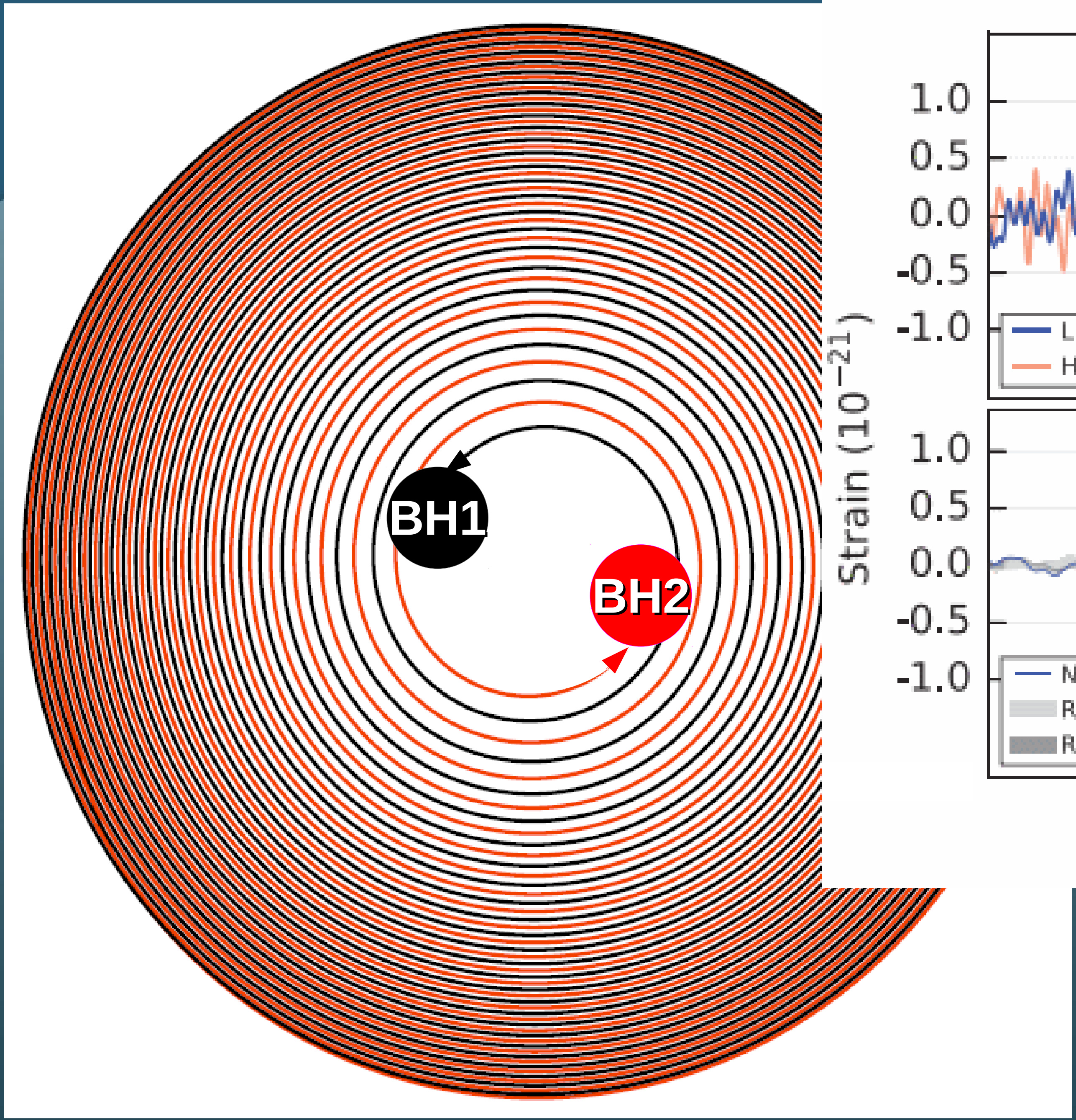
=

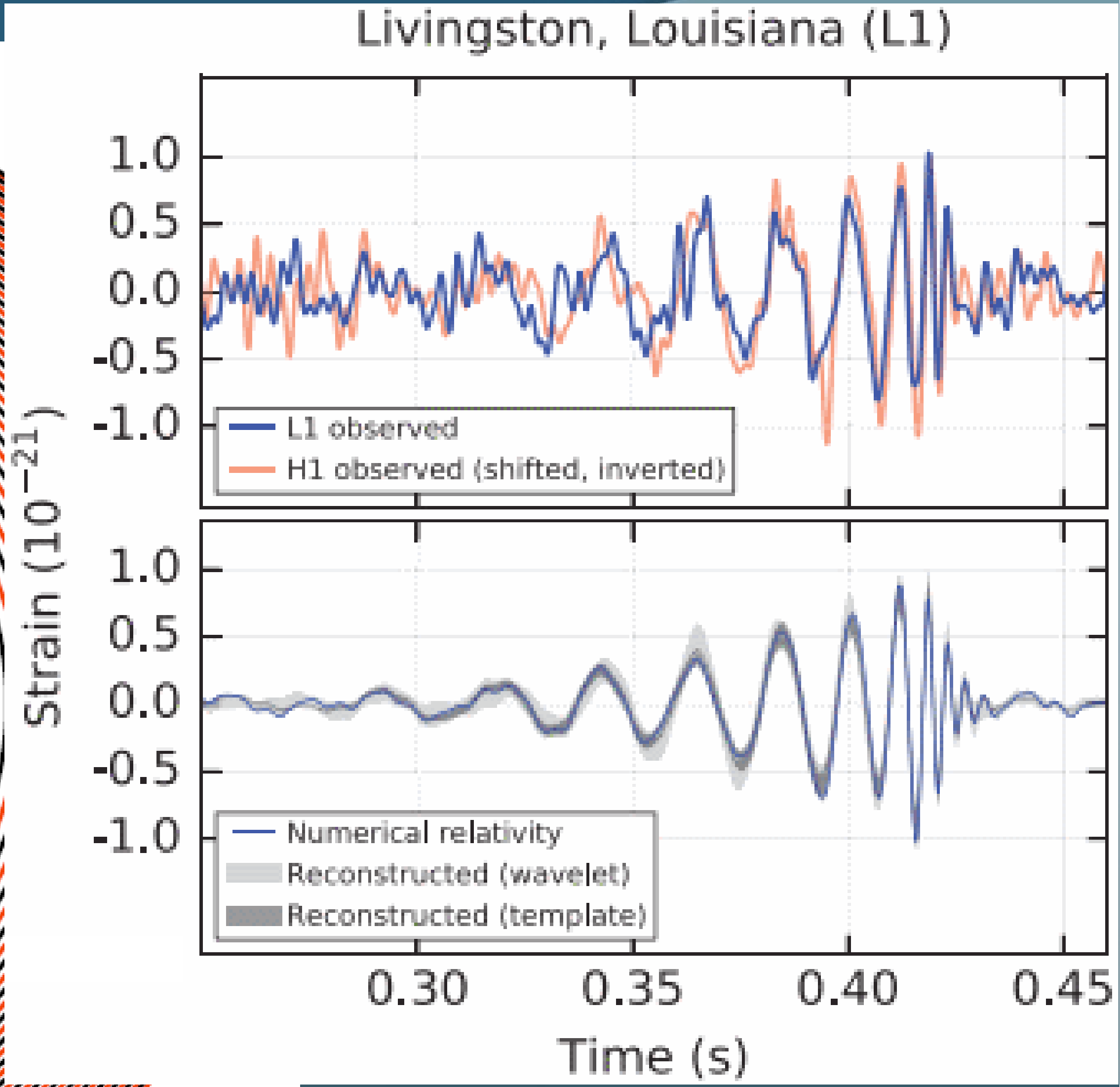



Livingston, Louisiana (L1)
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
0.30
0.35
0.40
0.45
Time (s)

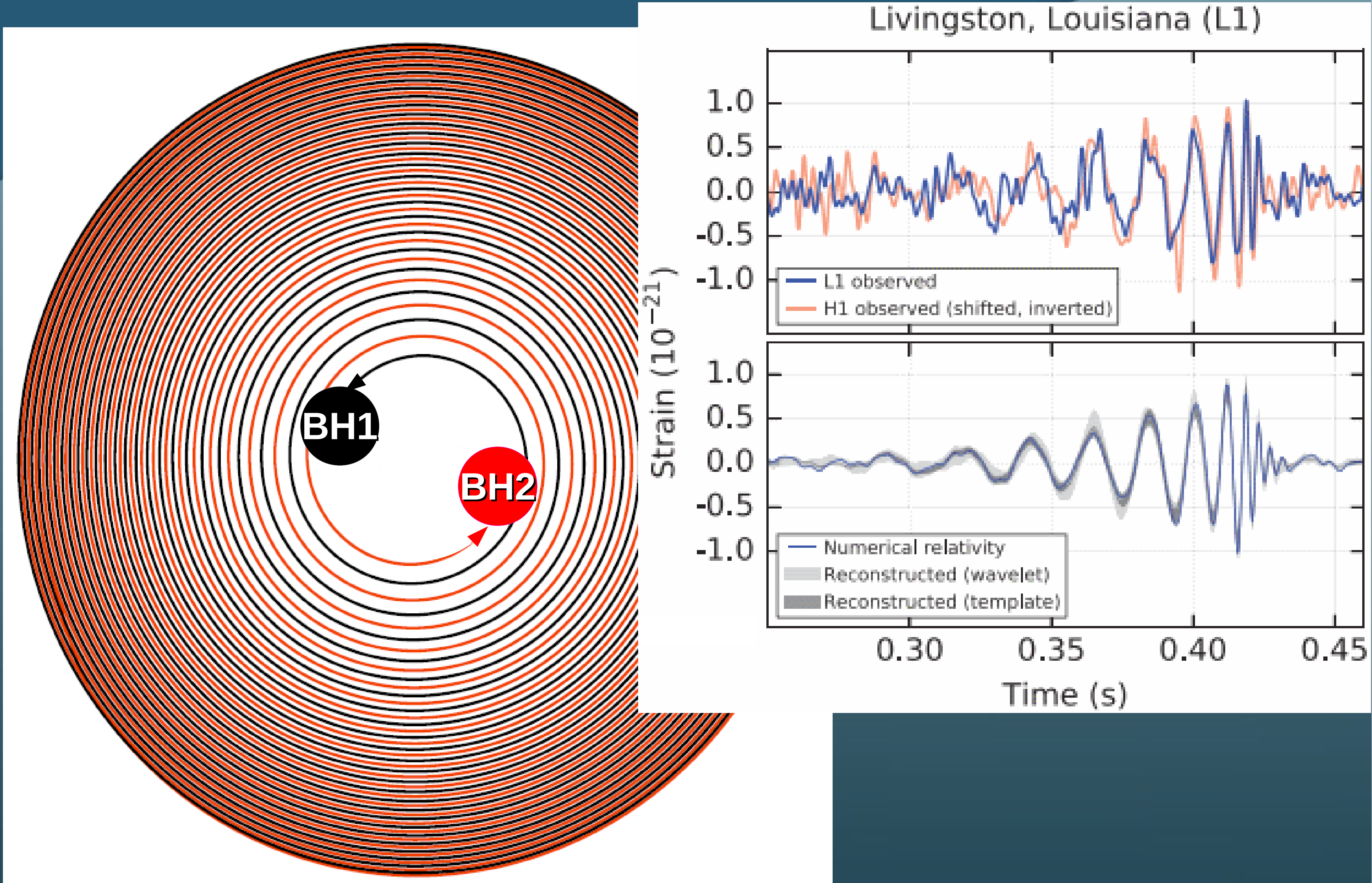



4 km





(original image:
Lovelace et al., CQG
29, 045003 (2012))

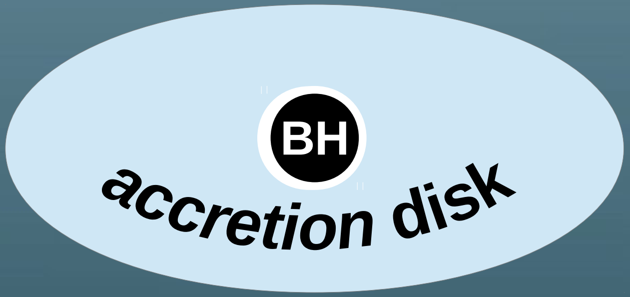


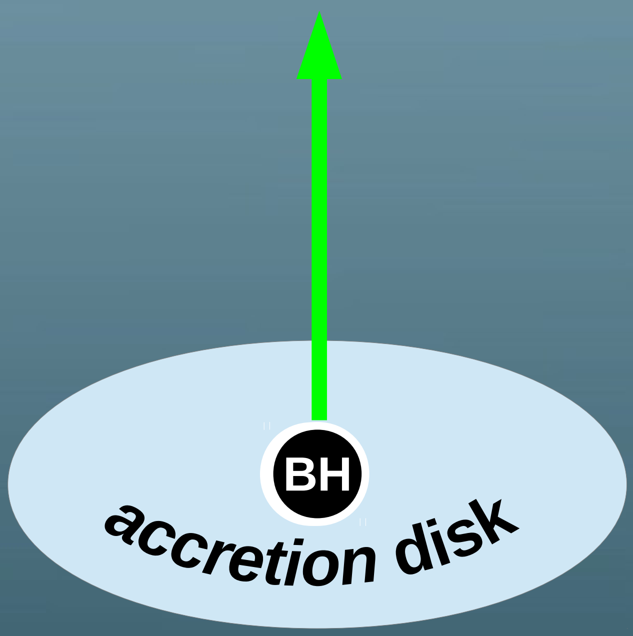
(original image:
Lovelace et al., CQG
29, 045003 (2012))

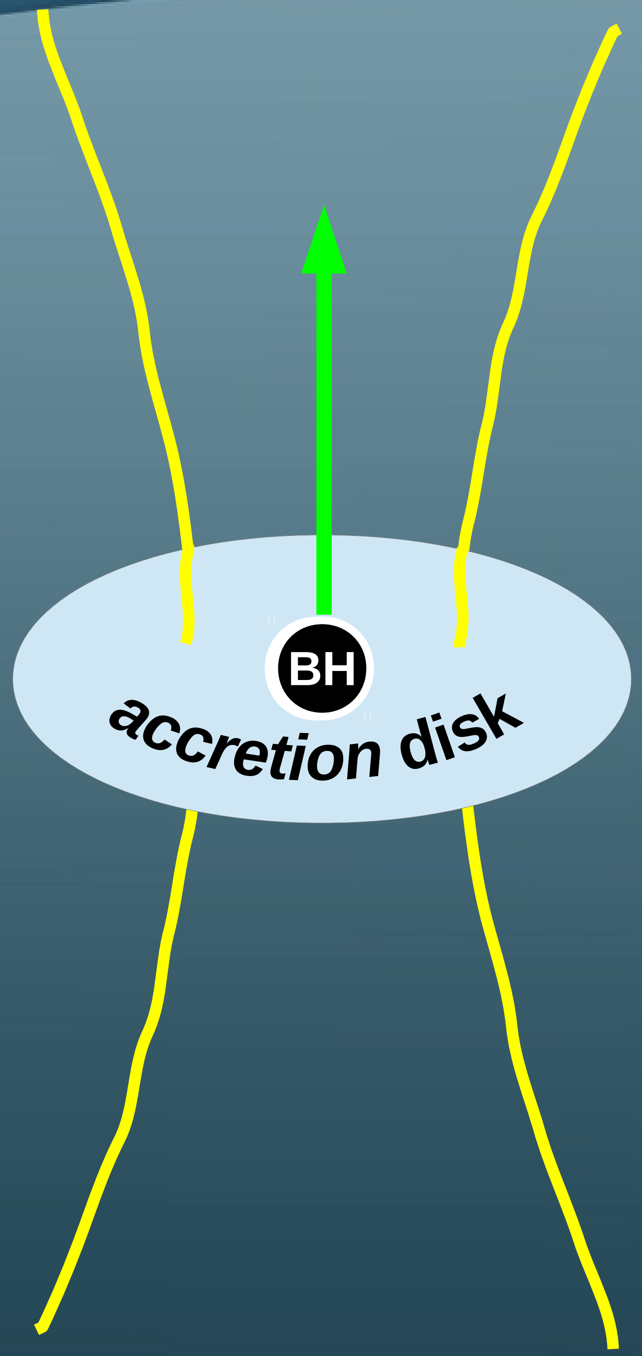
What remains to be seen?

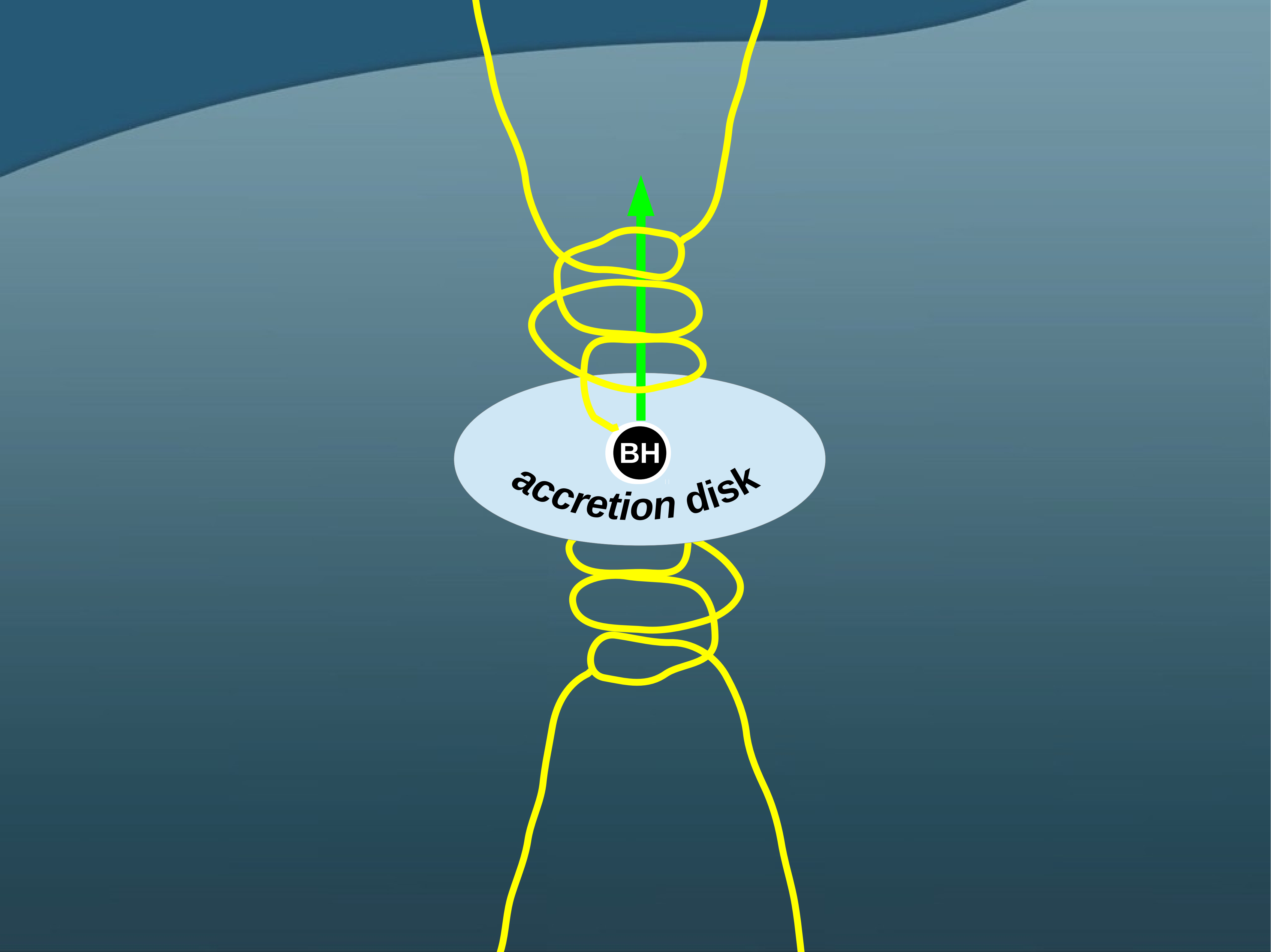

APEX, Chandra, MPG/ESO
AGN

AGN:
How does it work?


BH
accretion disk


BH
accretion disk

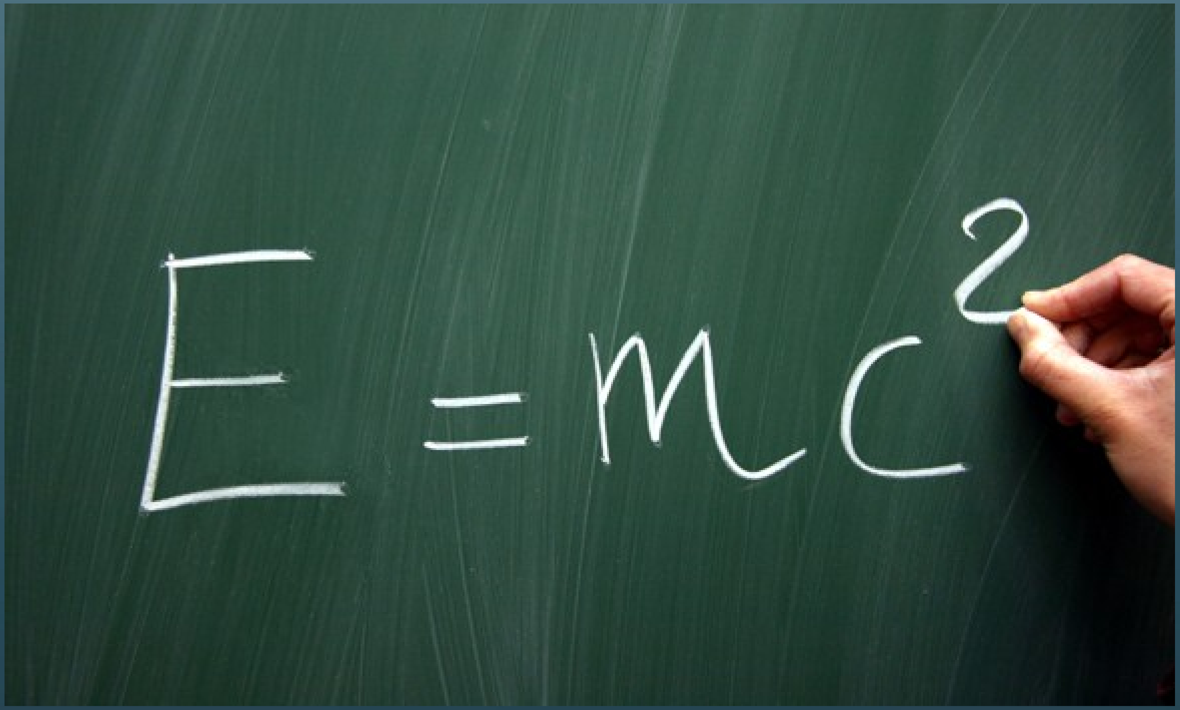

BH
accretion disk


BH
accretion disk


AGN

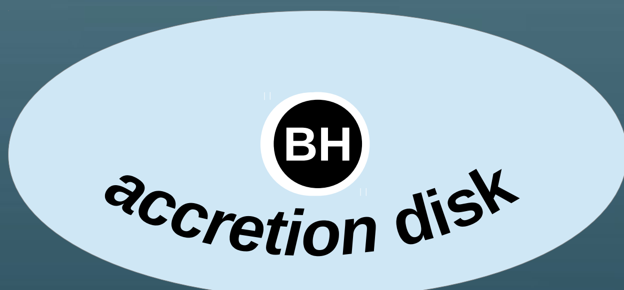
Mass-to-Energy Conversion Efficiency

- Typical stars: **~0.02%**
- Black hole accretion: **~10%**

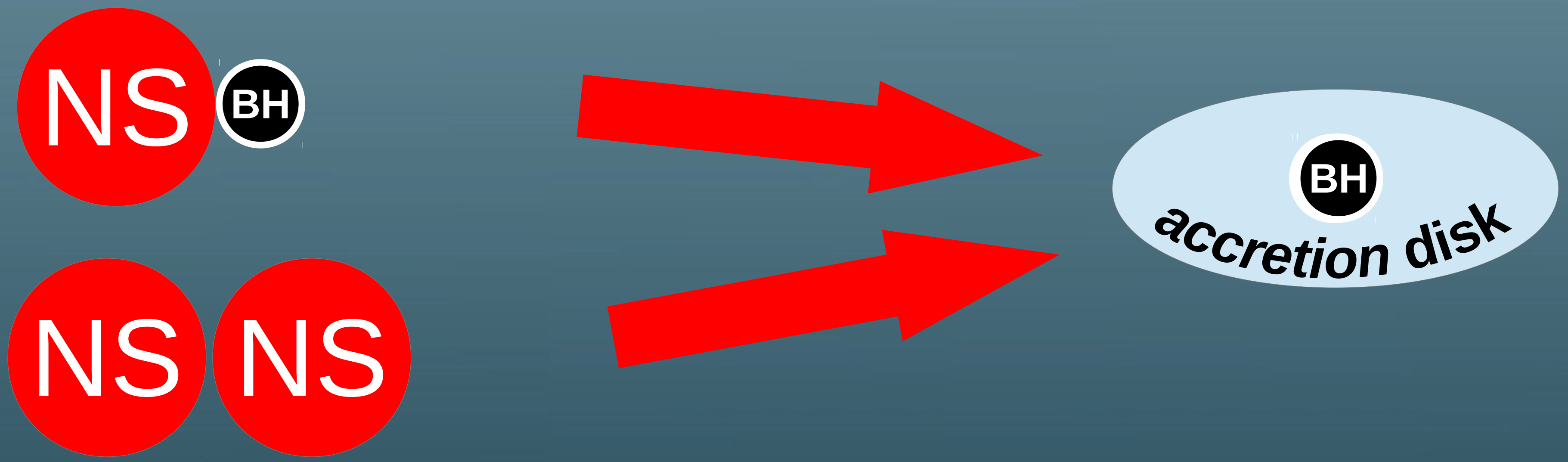


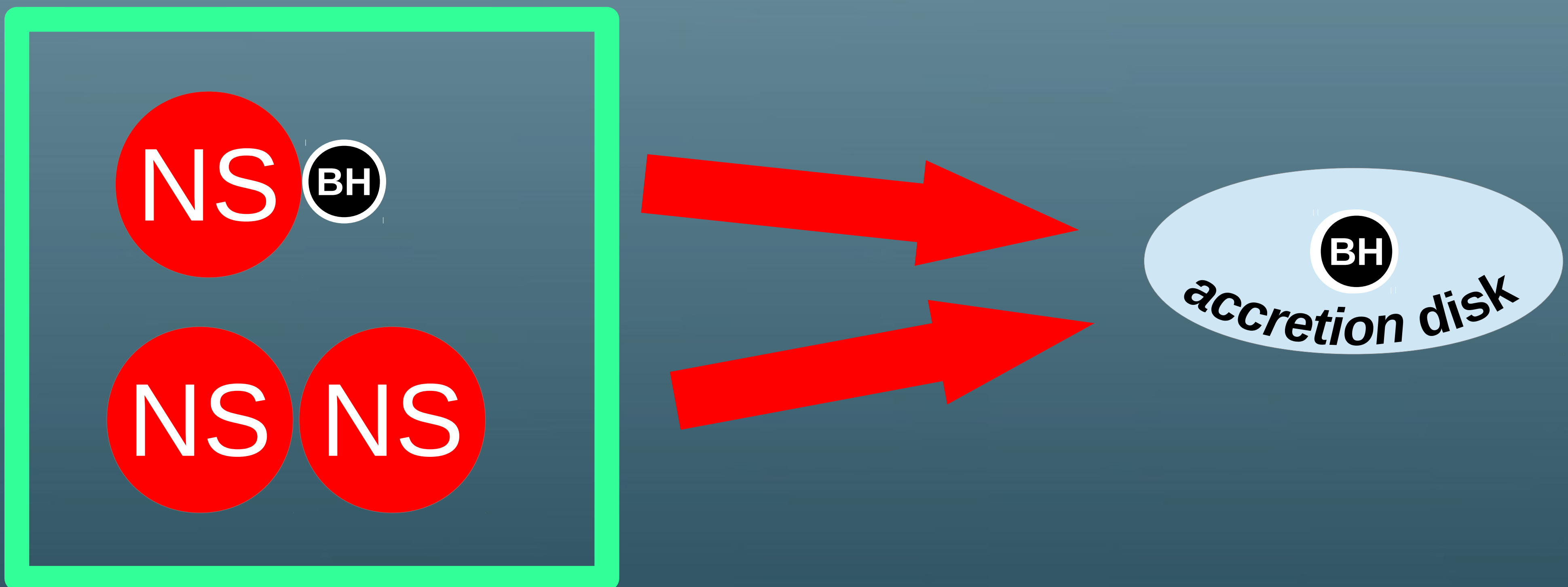
To get this enormously efficient release of EM energy

... required only a (spinning) BH+disk:

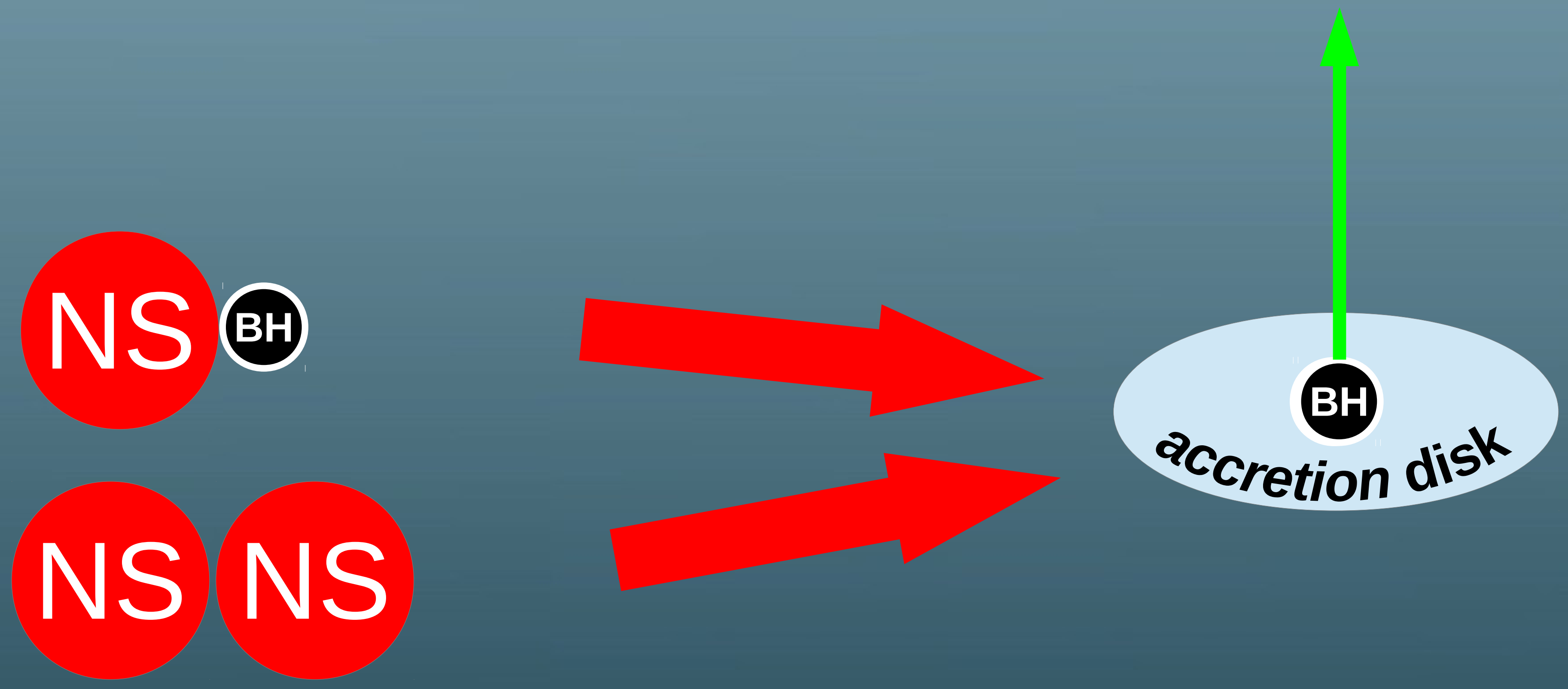


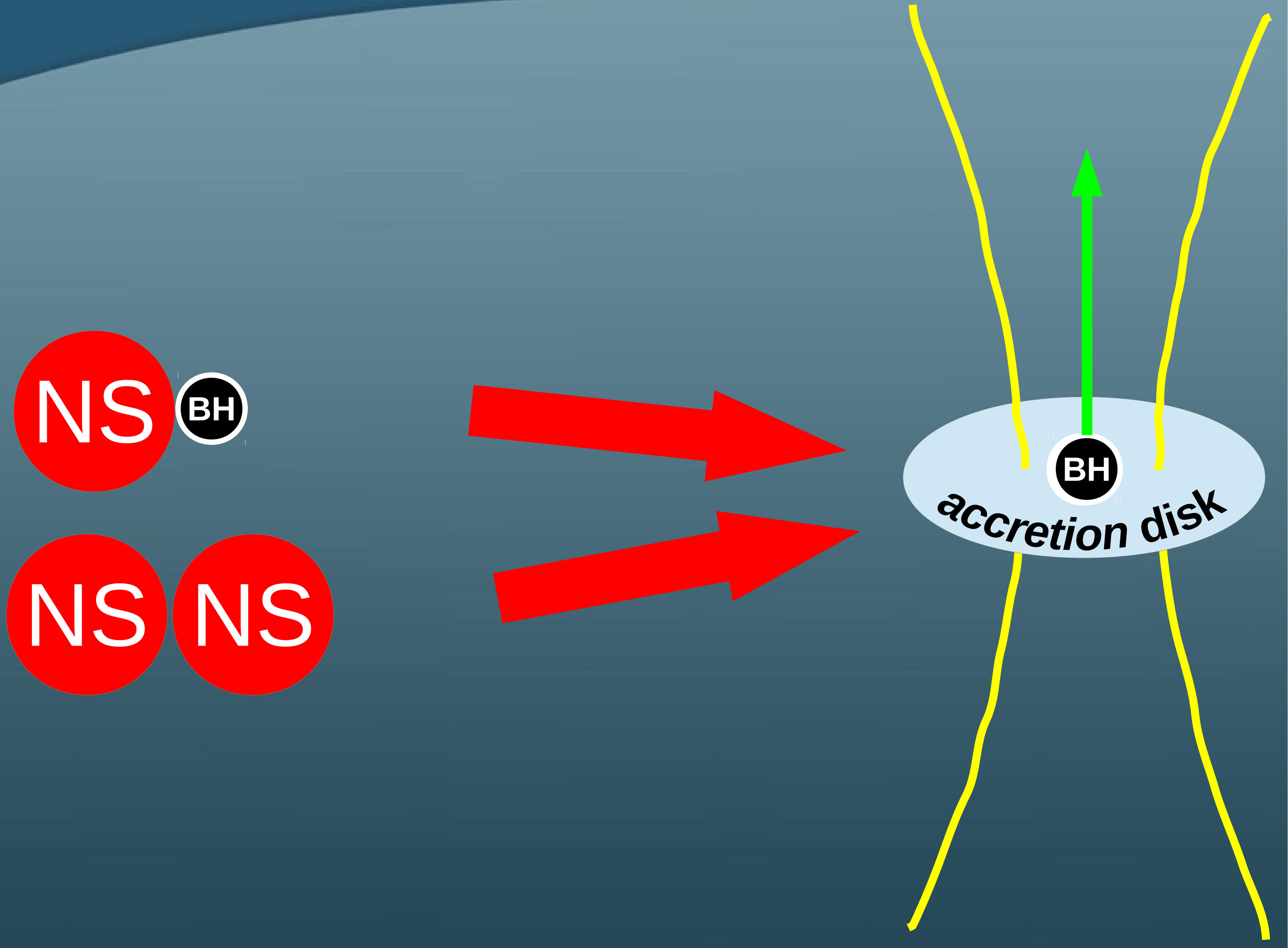
What other systems result in a BH+disk?

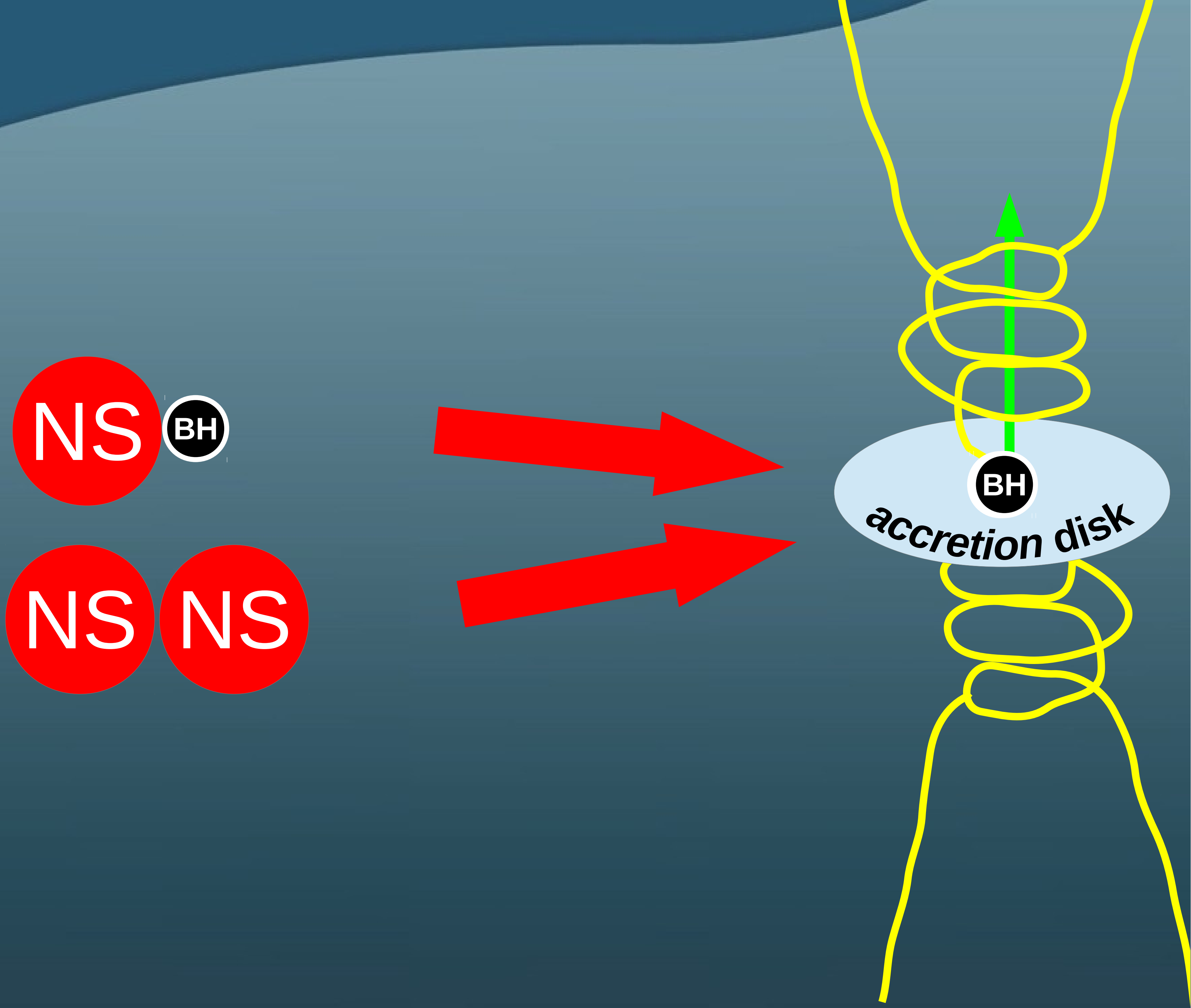

NS
BH
NS
NS
BH
accretion disk

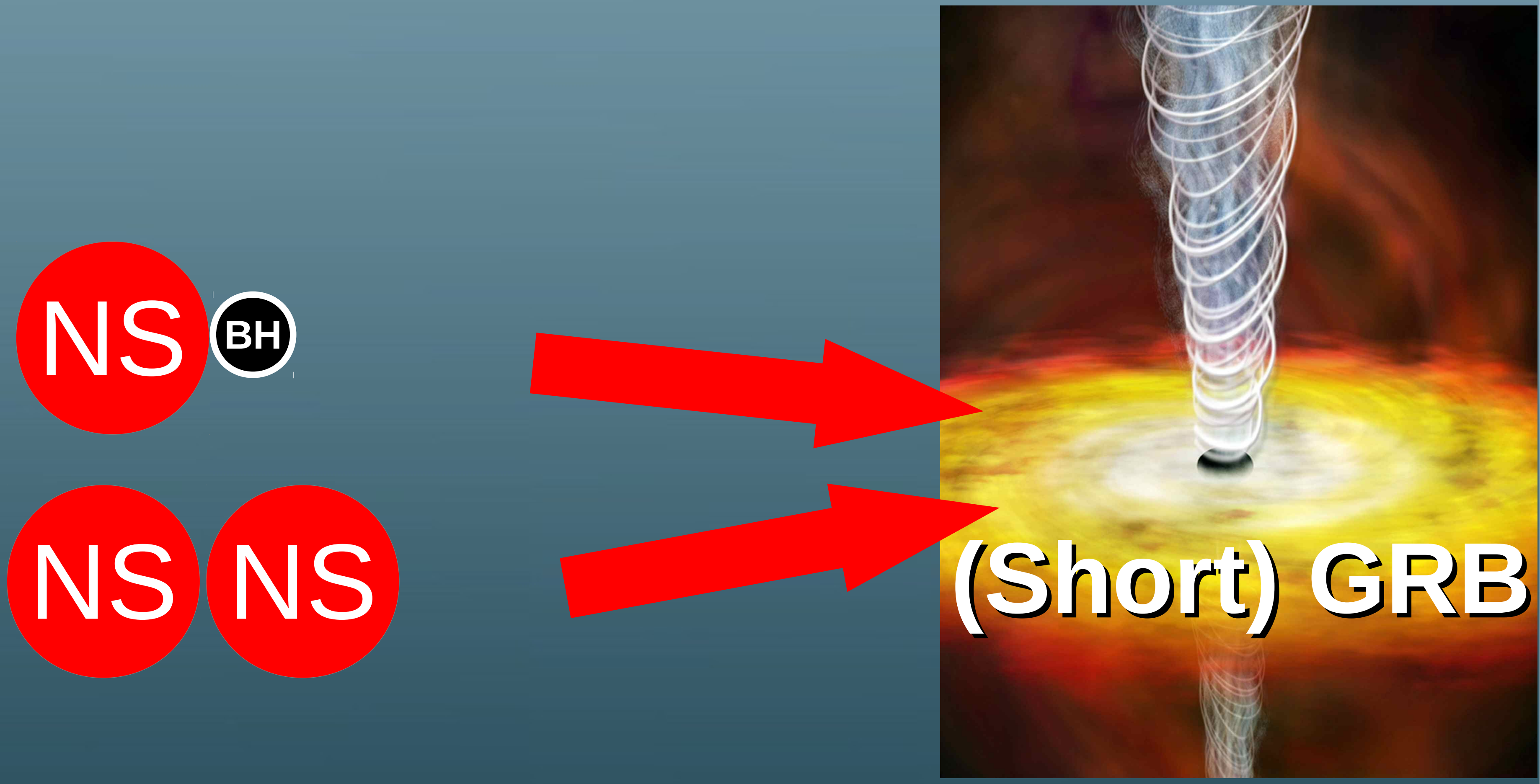


Detectable by LIGO!


NS
BH
NS
NS
BH
accretion disk


NS
BH
NS
NS
BH
accretion disk


NS
BH
NS
NS
BH
accretion disk


NS
BH
NS
NS
(Short) GRB

Short Gamma-Ray Bursts

- **Found in regions thought rich in NSs & BHs**
  - Host galaxy populated by older stars
    - → Most massive stars long dead, leaving behind NSs and BHs

Observationally consistent!

But what do our best theoretical models say?

Basic Equations

GR

Newtonian

Equations for gravitational field

$$G_{\mu\upsilon} = 8\pi T_{\mu\upsilon}$$

$$\nabla^2 \Phi = -4\pi\rho$$

Maxwell's equations in MHD limit

$$\partial_j\left(\sqrt{\gamma}B^j\right) = 0$$

$$\partial_t(\sqrt{\gamma}B^i) + \partial_j\left[\sqrt{\gamma}\left(\mathrm{v}^j B^i - \mathrm{v}^i B^j\right)\right] = 0$$

$$\nabla \cdot B = 0$$

$$\partial_t B = \nabla \times (v \times B)$$

Fluid equations

$$\partial_t \rho_* + \partial_j\left(\rho_* \mathrm{v}^j\right) = 0$$

$$\partial_t S_i + \partial_j\left(\alpha\sqrt{\gamma}T^j{}_i\right) = \frac{1}{2}\alpha\sqrt{\gamma}T^{\alpha\beta}\partial_i g_{\alpha\beta}$$

$$\partial_t \tau + \partial_j\left(-n_\mu \alpha\sqrt{\gamma}T^{\mu i} - \rho_* \mathrm{v}^j\right) = s$$

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

$$\rho\left(\partial_t v + v \cdot \nabla v\right) = -\nabla\left(P + \frac{B^2}{8\pi}\right) + \frac{B \cdot \nabla B}{4\pi} - \rho \nabla \Phi$$

$$\rho\left(\partial_t \varepsilon + v \cdot \nabla \varepsilon\right) + P \nabla \cdot v = 0$$

Basic Equations

GR | **Newtonian**

Equations for gravitational field

$$G_{\mu\upsilon} = 8\pi T_{\mu\upsilon}$$

$$\nabla^2 \Phi = -4\pi\rho$$

Maxwell's equations in MHD limit

$$\partial_j\left(\sqrt{\gamma}B^j\right) = 0$$

$$\partial_t(\sqrt{\gamma}B^i) + \partial_j\left[\sqrt{\gamma}\left(\mathrm{v}^j B^i - \mathrm{v}^i B^j\right)\right] = 0$$

$$\nabla \cdot B = 0$$

$$\partial_t B = \nabla \times (v \times B)$$

Fluid equations

$$\partial_t \rho_* + \partial_j\left(\rho_* \mathrm{v}^j\right) = 0$$

$$\partial_t S_i + \partial_j\left(\alpha\sqrt{\gamma}T^j{}_i\right) = \frac{1}{2}\alpha\sqrt{\gamma}T^{\alpha\beta}\partial_i g_{\alpha\beta}$$

$$\partial_t \tau + \partial_j\left(-n_\mu \alpha\sqrt{\gamma}T^{\mu i} - \rho_* \mathrm{v}^j\right) = s$$

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

$$\rho\left(\partial_t v + v \cdot \nabla v\right) = -\nabla\left(P + \frac{B^2}{8\pi}\right) + \frac{B \cdot \nabla B}{4\pi} - \rho \nabla \Phi$$

$$\rho\left(\partial_t \varepsilon + v \cdot \nabla \varepsilon\right) + P \nabla \cdot v = 0$$

Basic Equations

GR	Newtonian

Equations for gravitational field

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\nabla^2 \Phi = -4\pi\rho$$

Maxwell's equations in MHD limit

$$\partial_j\left(\sqrt{\gamma}B^j\right) = 0$$

$$\partial_t(\sqrt{\gamma}B^i) + \partial_j\left[\sqrt{\gamma}\left(\mathrm{v}^j B^i - \mathrm{v}^i B^j\right)\right] = 0$$

$$\nabla \cdot B = 0$$

$$\partial_t B = \nabla \times (v \times B)$$

Fluid equations

$$\partial_t \rho_* + \partial_j\left(\rho_* \mathrm{v}^j\right) = 0$$

$$\partial_t S_i + \partial_j\left(\alpha\sqrt{\gamma}T^j{}_i\right) = \frac{1}{2}\alpha\sqrt{\gamma}T^{\alpha\beta}\partial_i g_{\alpha\beta}$$

$$\partial_t \tau + \partial_j\left(-n_\mu \alpha\sqrt{\gamma}T^{\mu i} - \rho_* \mathrm{v}^j\right) = s$$

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

$$\rho\left(\partial_t v + v \cdot \nabla v\right) = -\nabla\left(P + \frac{B^2}{8\pi}\right) + \frac{B \cdot \nabla B}{4\pi} - \rho \nabla \Phi$$

$$\rho\left(\partial_t \varepsilon + v \cdot \nabla \varepsilon\right) + P \nabla \cdot v = 0$$

Basic Equations

GR | **Newtonian**

Equations for gravitational field

$$G_{\mu\upsilon} = 8\pi T_{\mu\upsilon}$$

$$\nabla^2 \Phi = -4\pi\rho$$

Maxwell's equations in MHD limit

$$\partial_j \left(\sqrt{\gamma} B^j \right) = 0$$

$$\partial_t (\sqrt{\gamma} B^i) + \partial_j \left[\sqrt{\gamma} \left(\mathrm{v}^j B^i - \mathrm{v}^i B^j \right) \right] = 0$$

$$\nabla \cdot B = 0$$

$$\partial_t B = \nabla \times (v \times B)$$

Fluid equations

$$\partial_t \rho_* + \partial_j \left(\rho_* \mathrm{v}^j \right) = 0$$

$$\partial_t S_i + \partial_j \left(\alpha \sqrt{\gamma} T^j{}_i \right) = \frac{1}{2} \alpha \sqrt{\gamma} T^{\alpha\beta} \partial_i g_{\alpha\beta}$$

$$\partial_t \tau + \partial_j \left(-n_\mu \alpha \sqrt{\gamma} T^{\mu i} - \rho_* \mathrm{v}^j \right) = s$$

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

$$\rho \left(\partial_t v + v \cdot \nabla v \right) = -\nabla \left(P + \frac{B^2}{8\pi} \right) + \frac{B \cdot \nabla B}{4\pi} - \rho \nabla \Phi$$

$$\rho \left(\partial_t \varepsilon + v \cdot \nabla \varepsilon \right) + P \nabla \cdot v = 0$$

Basic Equations

Missing Physics:
Proper magnetosphere modeling
(Ideal MHD eqs. become stiff in magnetospheres)
Neutrinos (no cooling)
Photons (no spectra)

$$\partial_j\left(\sqrt{\gamma}B^j\right)=0$$

$$\partial_t(\sqrt{\gamma}B^i)+\partial_j\left[\sqrt{\gamma}\left(\mathrm{v}^jB^i-\mathrm{v}^iB^j\right)\right]=0$$

$$\nabla\cdot B=0$$

$$\partial_t B=\nabla\times\left(v\times B\right)$$

Fluid equations

$$\partial_t\rho_*+\partial_j\left(\rho_*\mathrm{v}^j\right)=0$$

$$\partial_t S_i+\partial_j\left(\alpha\sqrt{\gamma}T^j{}_i\right)=\frac{1}{2}\alpha\sqrt{\gamma}T^{\alpha\beta}\partial_i g_{\alpha\beta}$$

$$\partial_t\tau+\partial_j\left(-n_\mu\alpha\sqrt{\gamma}T^{\mu i}-\rho_*\mathrm{v}^j\right)=s$$

$$\partial_t\rho+\nabla\cdot\left(\rho v\right)=0$$

$$\rho\left(\partial_t v+v\cdot\nabla v\right)=-\nabla\left(P+\frac{B^2}{8\pi}\right)+\frac{B\cdot\nabla B}{4\pi}-\rho\nabla\Phi$$

$$\rho\left(\partial_t\varepsilon+v\cdot\nabla\varepsilon\right)+P\nabla\cdot v=0$$

Basic Equations

Missing Physics:

Proper magnetosphere modeling

(Ideal MHD eqs. become stiff in magnetospheres)

Neutrinos (no cooling)

Photons (no spectra)

$$\partial_j\left(\sqrt{\gamma}B^j\right)=0$$

$$\partial_t(\sqrt{\gamma}B^i)+\partial_j\left[\sqrt{\gamma}\left(\mathrm{v}^jB^i-\mathrm{v}^iB^j\right)\right]=0$$

$$\nabla\cdot B=0$$

$$\partial_t B=\nabla\times(v\times B)$$

Fluid equations

$$\partial_t\rho_*+\partial_j\left(\rho_*\mathrm{v}^j\right)=0$$

$$\partial_t S_i+\partial_j\left(\alpha\sqrt{\gamma}T^j{}_i\right)=\frac{1}{2}\alpha\sqrt{\gamma}T^{\alpha\beta}\partial_i g_{\alpha\beta}$$

$$\partial_t\tau+\partial_j\left(-n_\mu\alpha\sqrt{\gamma}T^{\mu i}-\rho_*\mathrm{v}^j\right)=s$$

$$\partial_t\rho+\nabla\cdot(\rho v)=0$$

$$\rho\left(\partial_t v+v\cdot\nabla v\right)=-\nabla\left(P+\frac{B^2}{8\pi}\right)+\frac{B\cdot\nabla B}{4\pi}-\rho\nabla\Phi$$

$$\rho\left(\partial_t\varepsilon+v\cdot\nabla\varepsilon\right)+P\nabla\cdot v=0$$

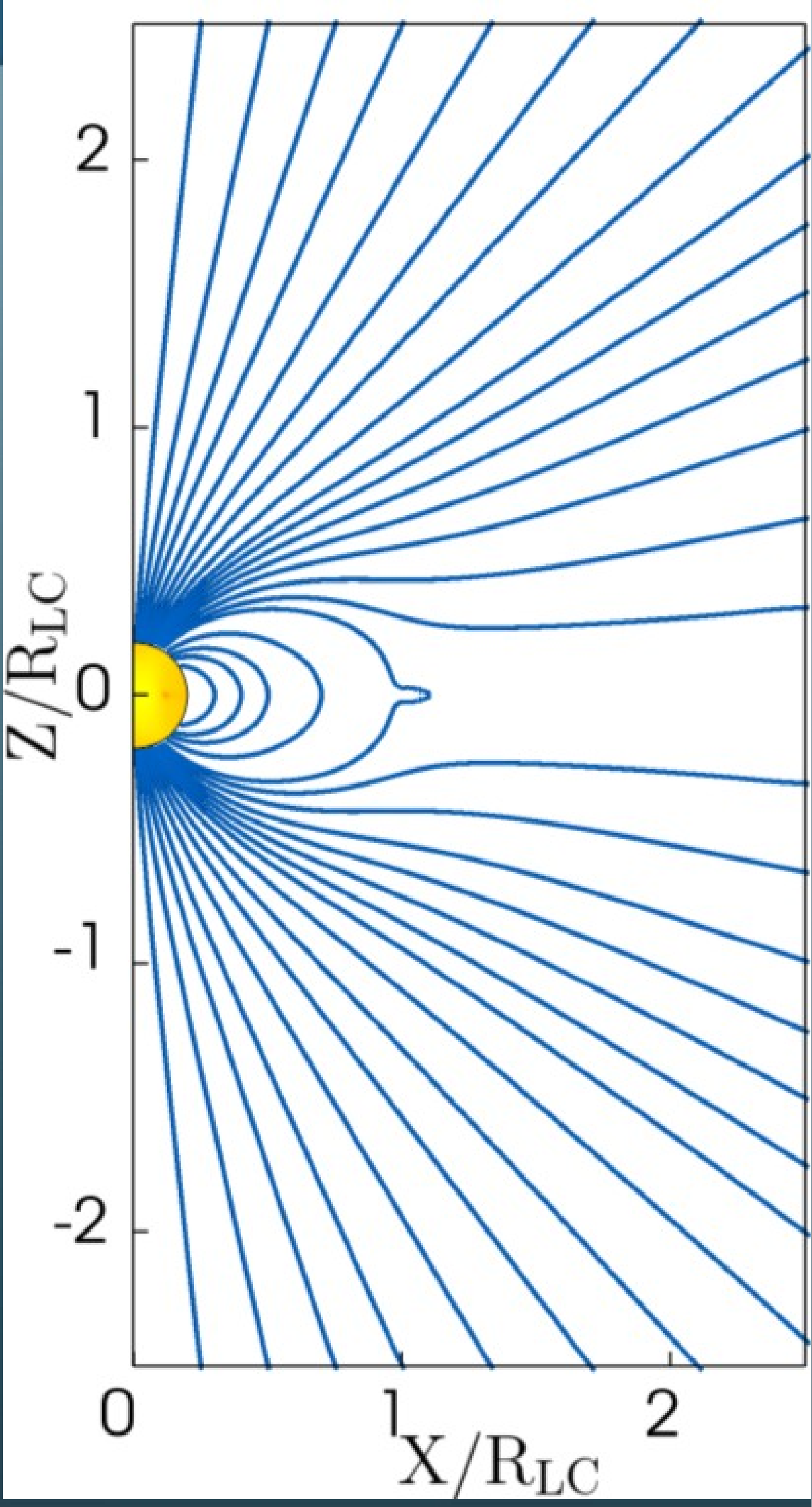
GiRaFFE

General Relativistic Force-Free Electrodynamics

- Gravitational waves (GWs) drive black holes and neutron stars to inspiral and merge
  - → Exterior magnetic fields change rapidly
  - → Possible EM counterpart to GW signal!
- Such EM counterparts could yield deep insights into these extreme objects
  - But we need firmer theoretical foundation for modeling them
- **GiRaFFE**: Solves equations of general relativistic force-free electrodynamics (GRFFE), needed to realistically model such counterparts

GiRaFFE
Results: Simple pulsar model

Starting with initial dipole field, magnetic field lines (blue) open at the light cylinder (X/R_LC = 1), due to rotation of magnetized star (yellow)



Basic Equations

Missing Physics:

Proper magnetosphere modeling

(Ideal MHD eqs. become stiff in magnetospheres)

Neutrinos (no cooling)

Photons (no spectra)

$\partial_t(\sqrt{\gamma}$

Why not add everything now?

Short answer: Current simulations are extremely computationally expensive, and CPUs are not getting faster

(Moore's Law has ended)

$\partial_t \rho_* + \partial$

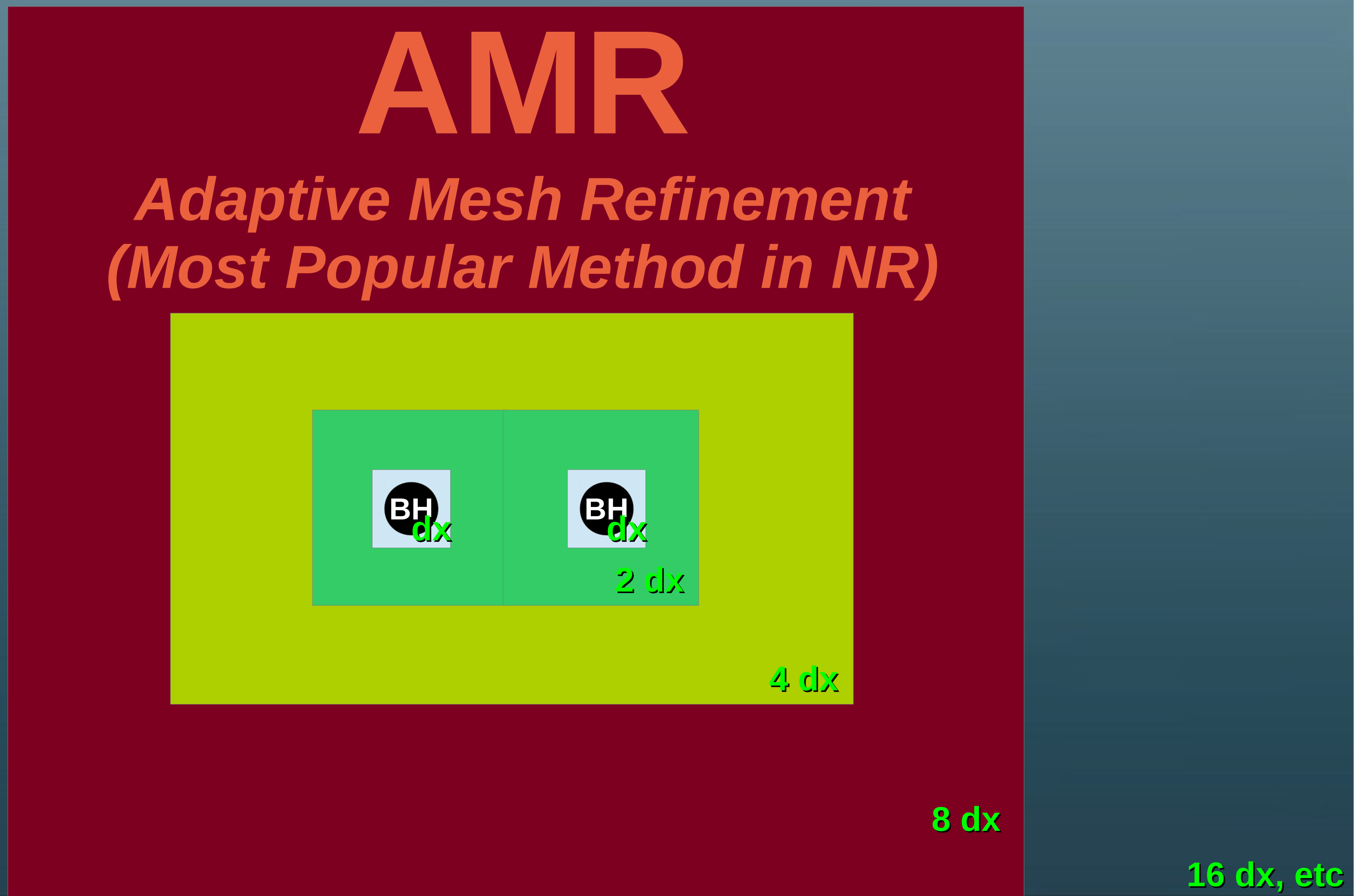
→ Can't add new physics without greatly improving efficiency

$\partial_t S_i + \partial$ … B $- \rho \nabla \Phi$

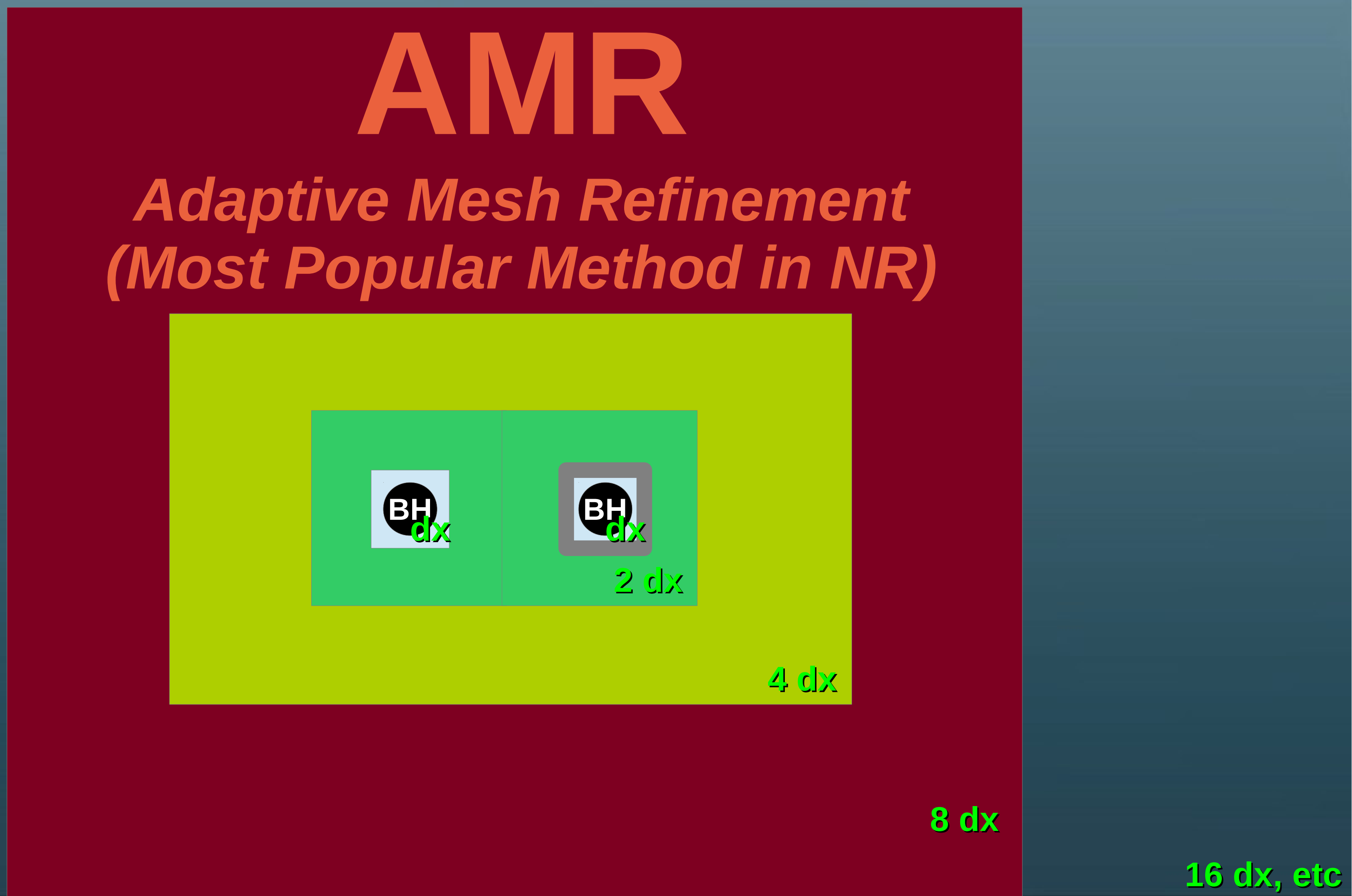
The future lies in developing more efficient algorithms

$\partial_t \tau + \partial_j$

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations



Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

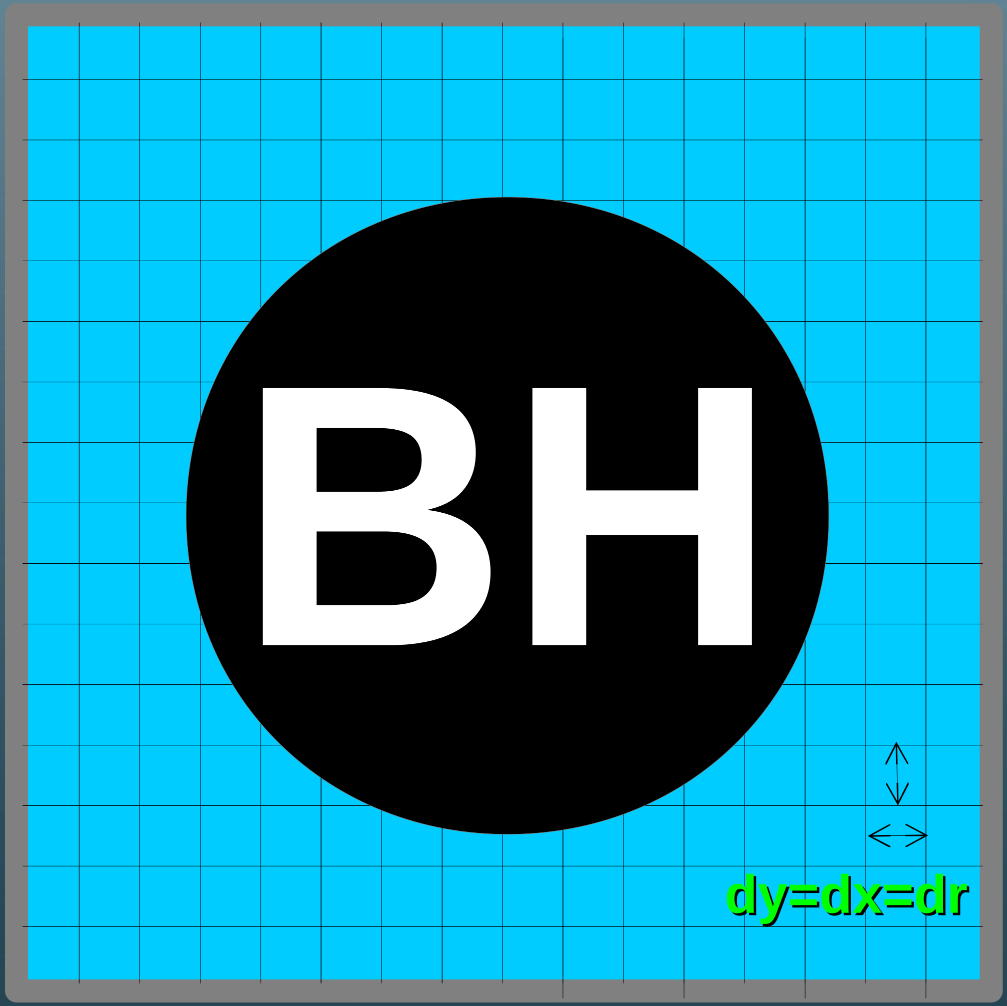


Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

Near-Spherical Object

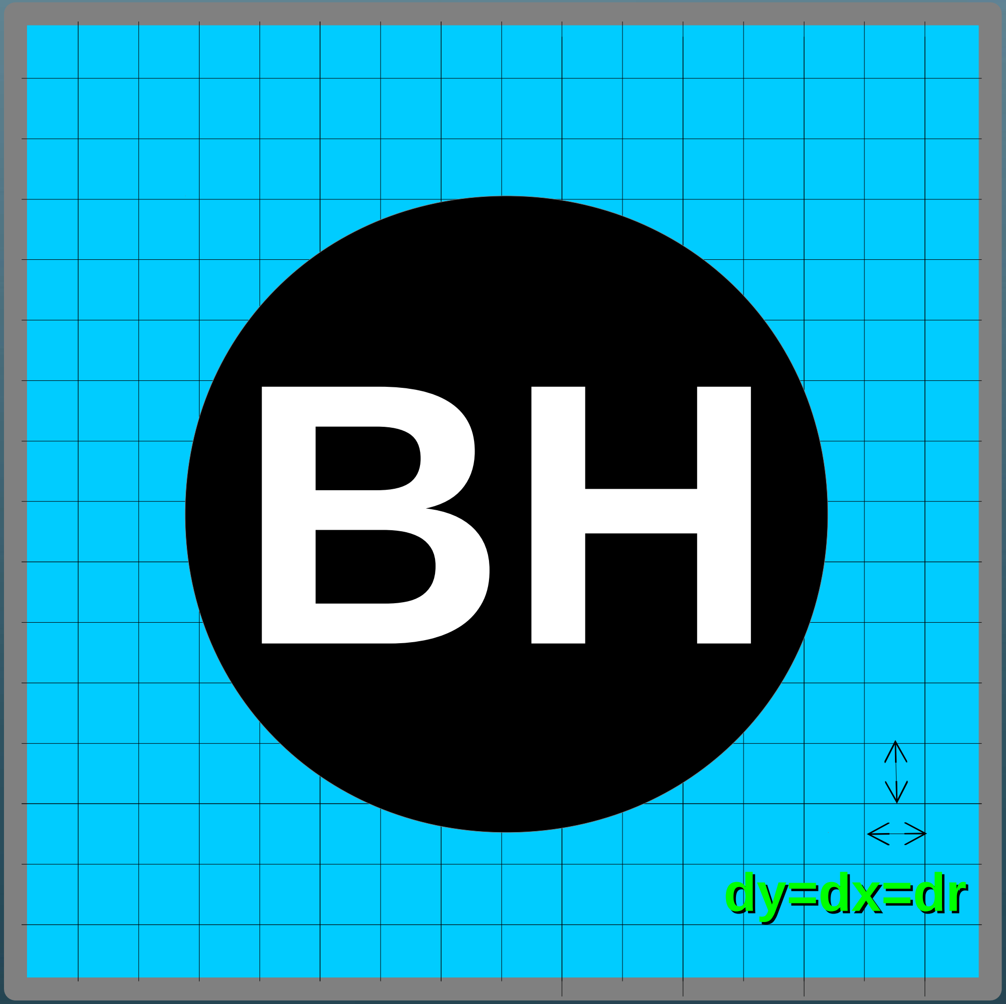
- **Highest res needed in radial dirn, need 1/3—1/10 points in angular directions Cost: Nr*Ntheta*Nphi ~ 1/100 Nr^3 → 1/10 Nr^3**
- **Cartesian grid: need dx=dy=dz=dr. Cost: Nx*Ny*Nz ~ Nr^3**
- **So far, spherical polar grid ~ 10-100x more efficient than Cartesian**



Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

Near-Spherical Object

- Highest res needed in radial dirn, need 1/3—1/10 points in angular directions
  Cost:
  Nr*Ntheta*Nphi ~ 1/100 Nr^3 → 1/10 Nr^3
- Cartesian grid: need dx=dy=dz=dr.
  Cost:
  Nx*Ny*Nz ~ Nr^3
- So far, spherical polar grid ~ 10-100x more efficient than Cartesian



What about dr along diagonal?

- Cube diagonal = √3*sidelength → to get dr resolution in all directions, need to reduce dx,dy,dz by √3
- Since cost in memory ~1/dx^3, "fitting the round peg in a square hole" increases cost by another factor of $(\sqrt{3})^3$~5x!

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

Near-Spherical Object

- Highest res needed in radial dirn, need 1/3—1/10 points in angular directions Cost: **Nr*Ntheta*Nphi ~ 1/100 $Nr^3 \rightarrow$ 1/10 Nr^3**
- Cartesian grid: need dx=dy=dz=dr. Cost: **Nx*Ny*Nz ~ Nr^3**
- So far, spherical polar grid ~ 10-100x more efficient than Cartesian

Inefficiencies so far: ~50-500x

What about dr along diagonal?

- Cube diagonal = √3*sidelength → to get dr resolution in all directions, need to reduce dx,dy,dz by √3
- Since cost in memory ~$1/dx^3$, “fitting the round peg in a square hole” increases cost by another factor of **$(\sqrt{3})^3$~5x!**

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

AMR Box Boundary is a Cube...

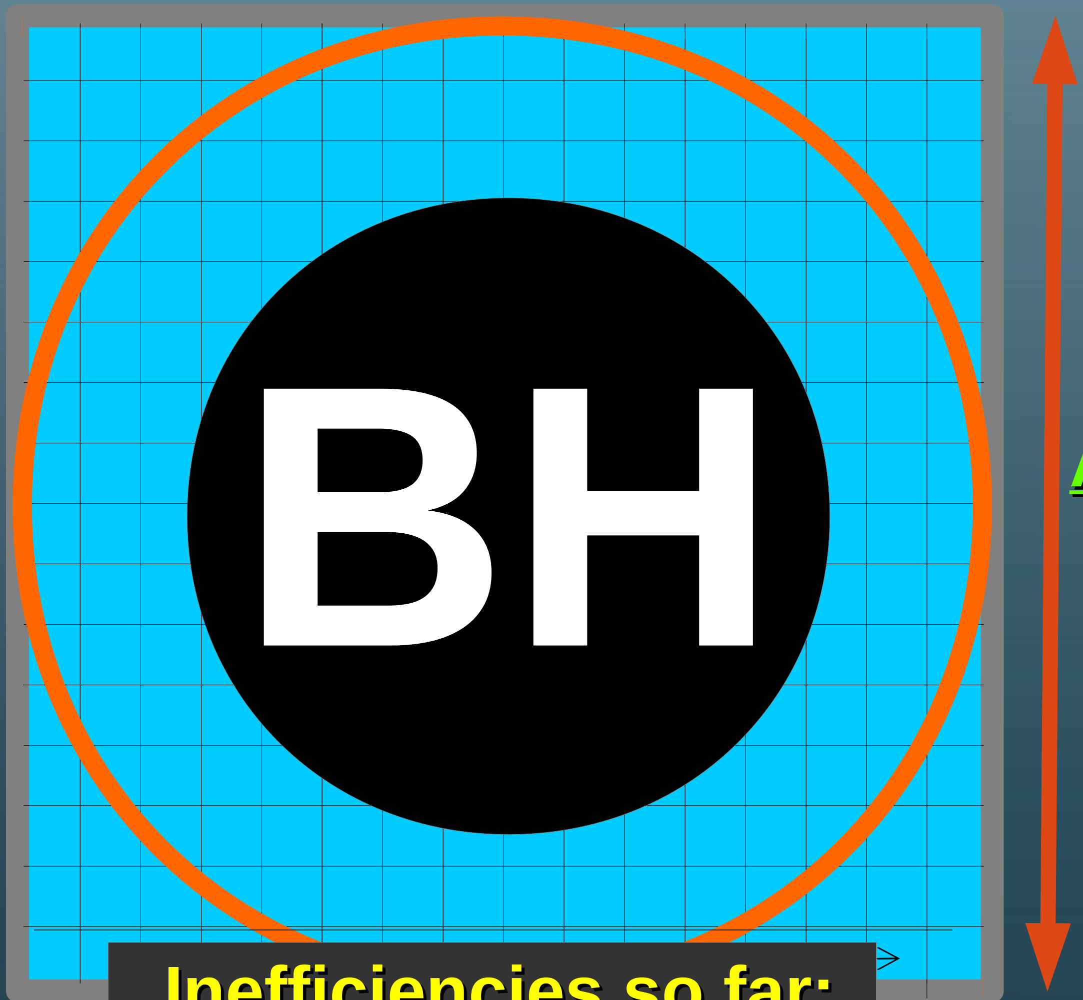
- … but fields fall off radially!
- → region outside orange circle is over-resolved by 2x
- Total volume of over-resolved region = 8-4/3 pi ~ 3.8 = about half the cube!
- So we gain by about another factor of 1.9x.

AMR Box side-length = 2

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

AMR Box Boundary is a Cube...

- ... but fields fall off radially!
- → region outside orange circle is over-resolved by 2x
- Total volume of over-resolved region = 8-4/3 pi ~ 3.8 = about half the cube!
- So we gain by about another factor of 1.9x.



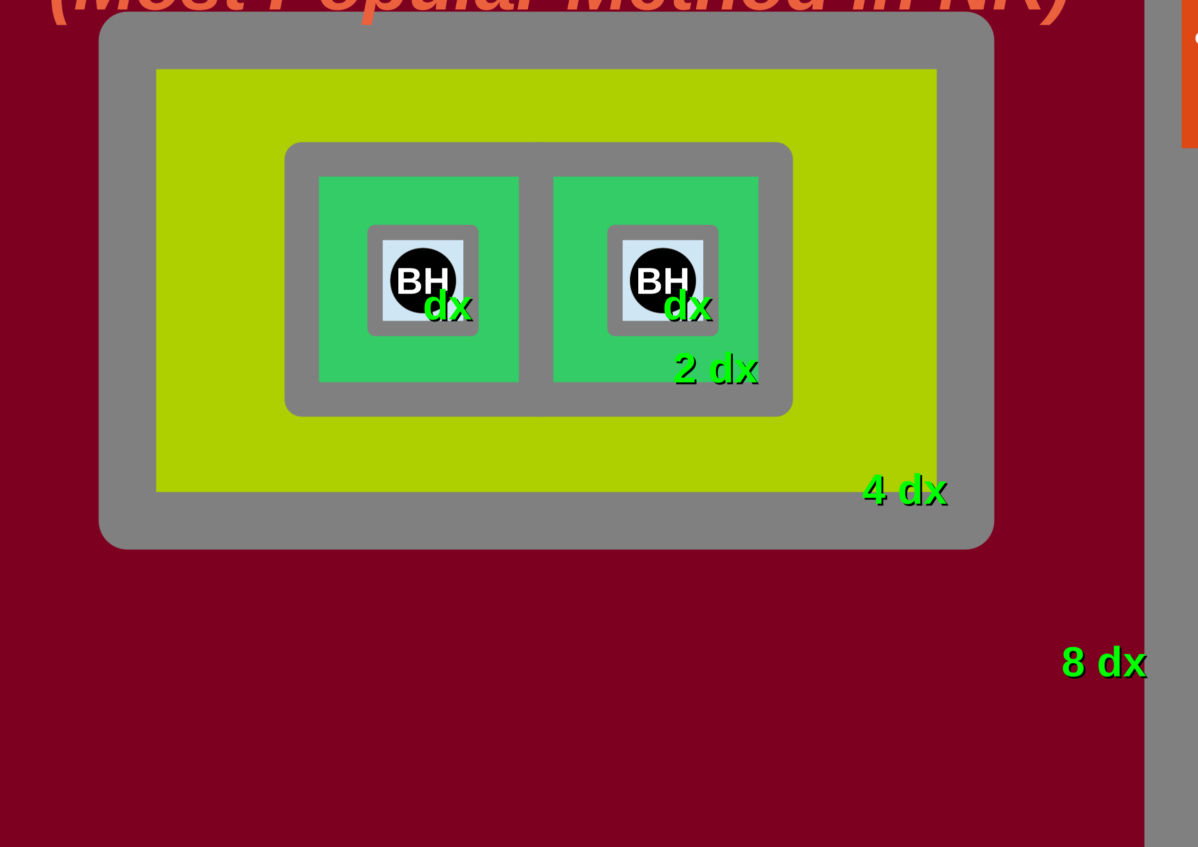
AMR Box side-length = 2

Inefficiencies so far: ~100-1,000x

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

AMR

Adaptive Mesh Refinement (Most Popular Method in NR)



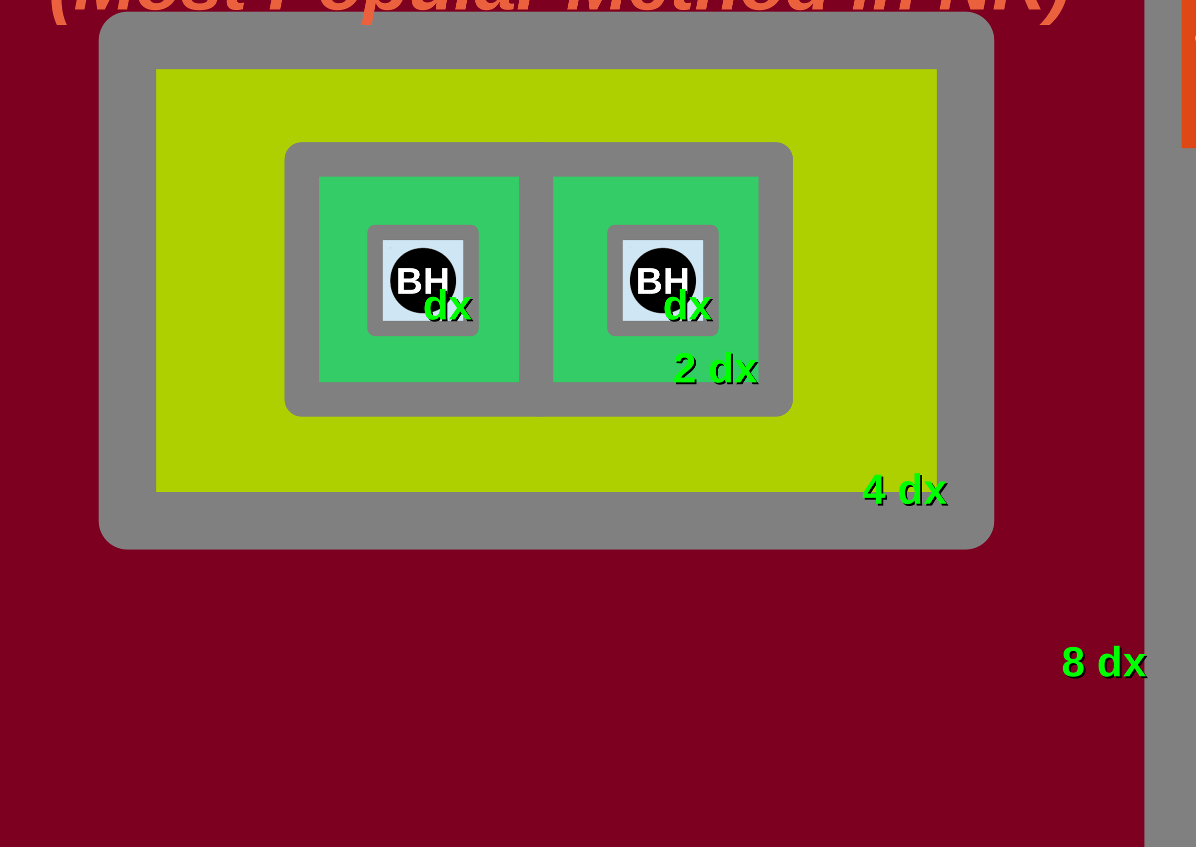
AMR

- Information must be interpolated across refinement boundaries.
- Interpolation → grids must overlap
- Overlap regions (grey) can take up 50% of overall computational domain!

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

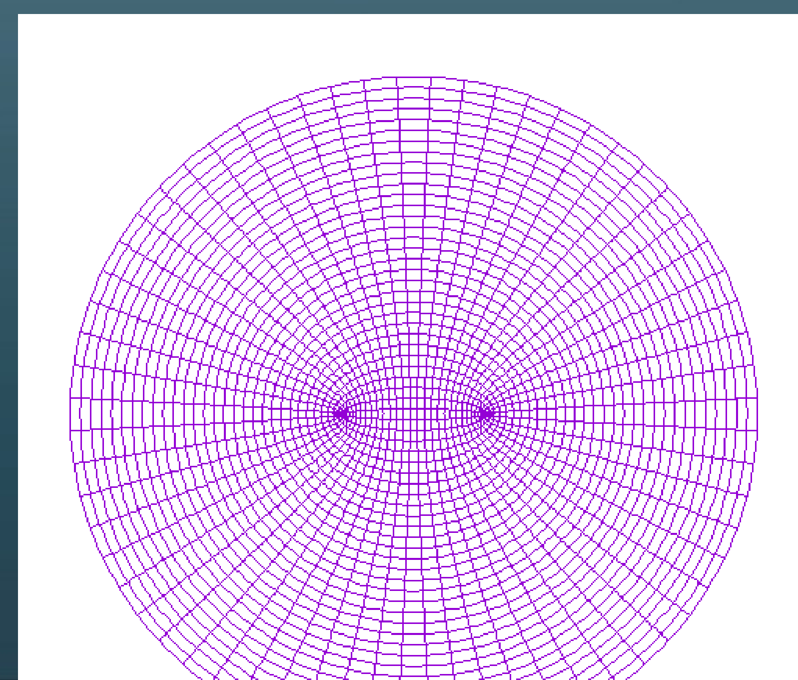
AMR

Adaptive Mesh Refinement (Most Popular Method in NR)



AMR

- **Information must be interpolated across refinement boundaries.**
- **Interpolation → grids must overlap**
- **Overlap regions (grey) can take up 50% of overall computational domain!**



Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

AMR Box Boundary is a Cube...

- … but fields fall off radially!
- →region outside orange circle is over-resolved by 2x
- Total volume of over-resolved region = 8-4/3 pi ~ 3.8 = about half the cube!
- So we gain by about another factor of 1.9x.

High-order finite difference with AMR

- → Enormous number of ghost zones at refinement boundaries!
- Ghost zones can take up 50% of overall computational domain!
- Bispherical coordinate system: Gain another ~2x

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

AMR Box Boundary is a Cube...

- … but fields fall off radially!
- →region outside orange circle is over-resolved by 2x
- Total volume of over-resolved region = 8-4/3 pi ~ 3.8 = about half the cube!
- So we gain by about another factor of 1.9x.

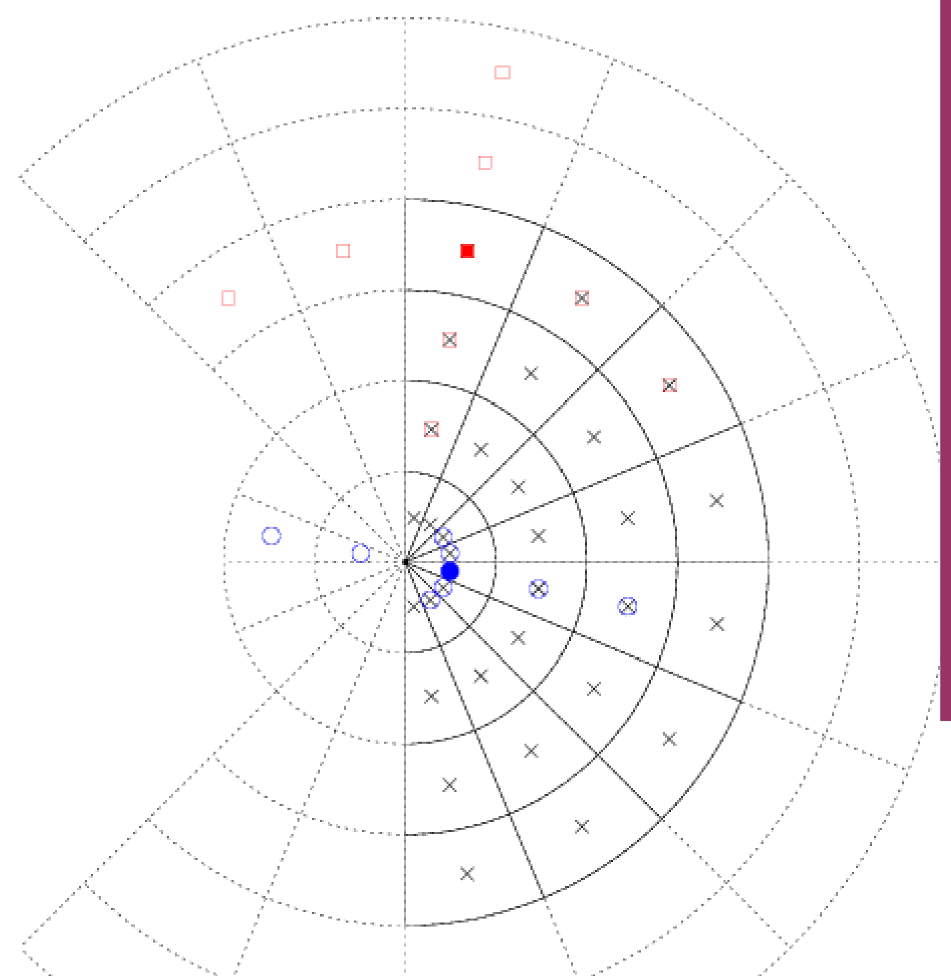
High-order finite difference with AMR

- → Enormous number of ghost zones at refinement boundaries!
- Ghost zones can take up 50% of overall computational domain!
- Bispherical coordinate system: Gain another ~2x

Inefficiencies: ~200-2,000x

Idea: Move to Spherical Polar Coordinates!

Cartesian Coordinates:
- Advantage:
  - Well-behaved numerically
- Disadvantage:
  - ~200—2,000x inefficient in computational cost, memory overhead

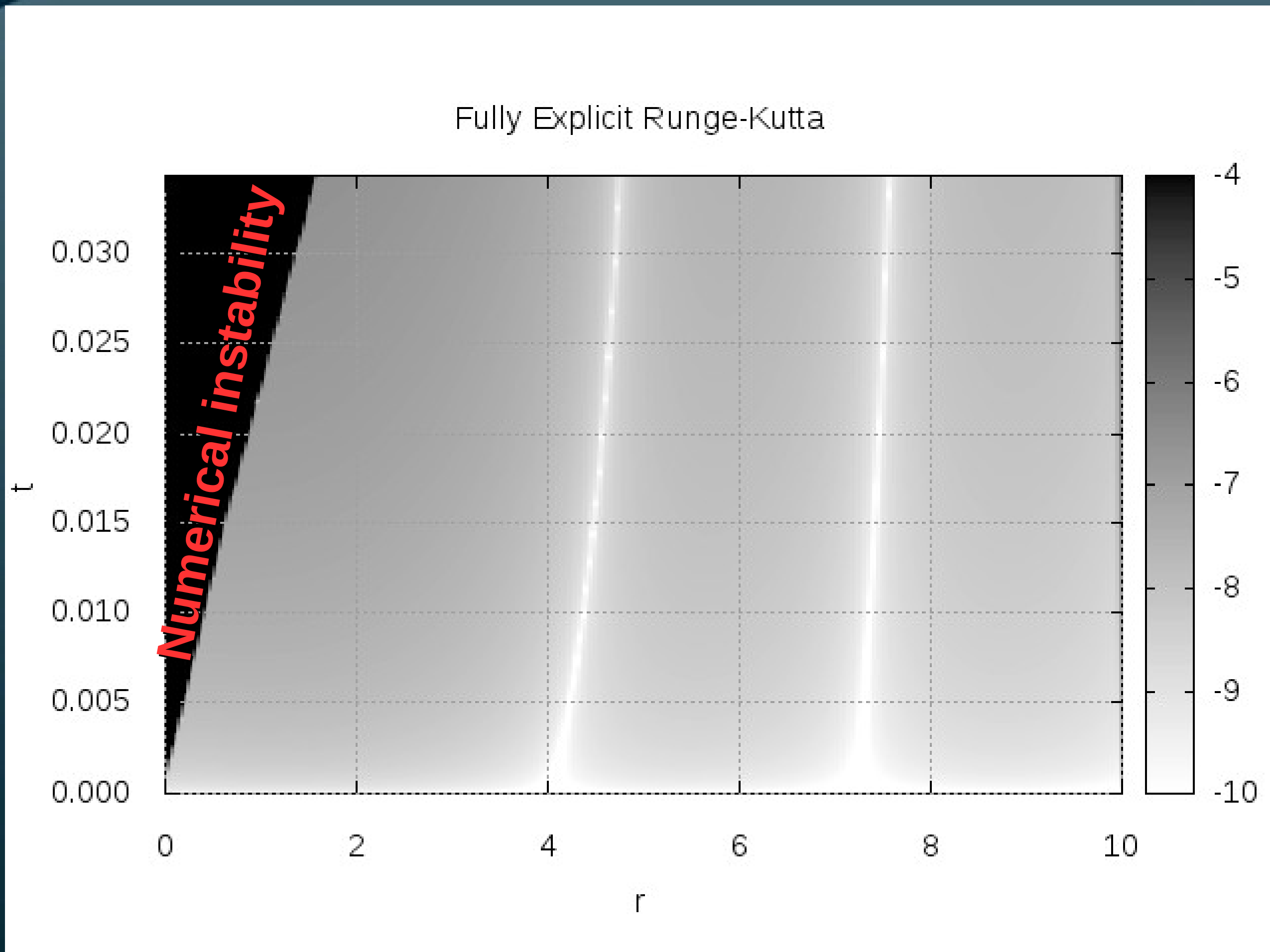


Baumgarte et al., Phys. Rev. D 87, 044026 (2012)
arXiv:1211.6632

Spherical Polar Coordinates:
- Advantage:
  - Very inexpensive computationally!
- Disadvantage:
  - Stability issues?

  Recent breakthroughs address stability issues!

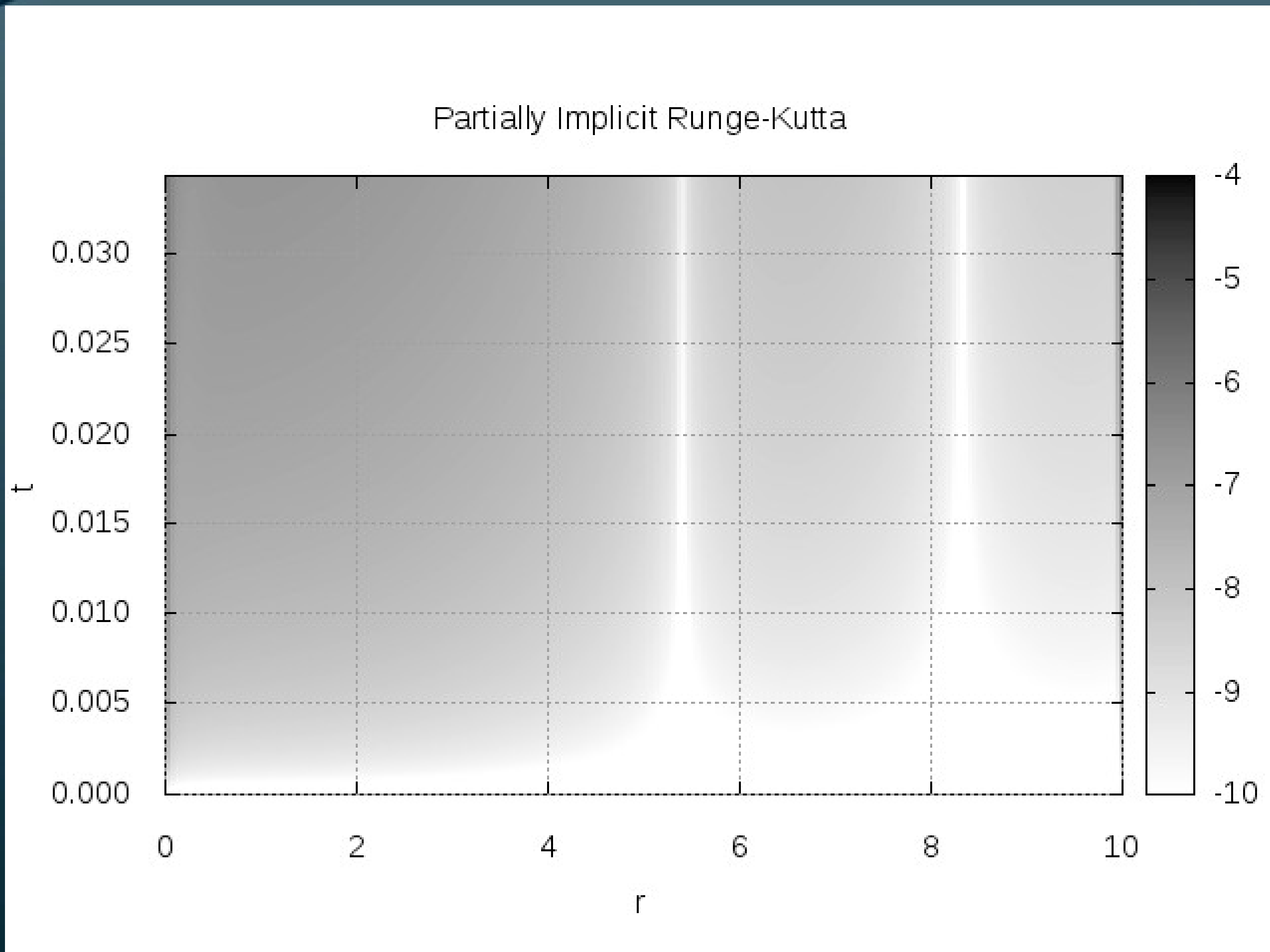
Idea: Move to Spherical Polar Coordinates!



$$\partial_t^2 u = \partial_r^2 u + \frac{2}{r}\partial_r u$$

<ins>Coordinate singularities</ins> lead to instabilities in traditional numerical schemes (e.g., 1+1 spherical scalar wave in RK2)

Idea: Move to Spherical Polar Coordinates!



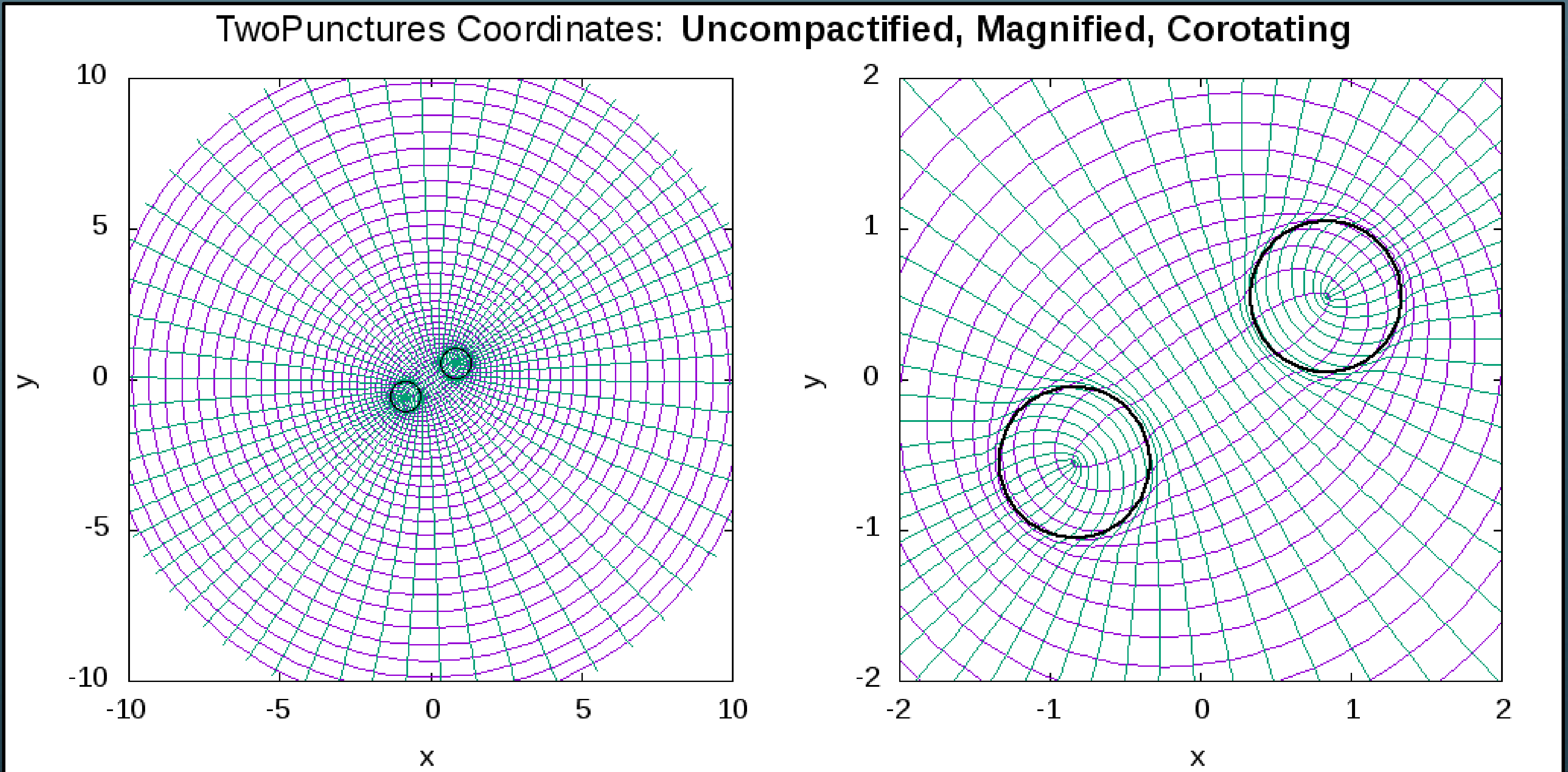
$$\partial_t^2 u = \partial_r^2 u + \boxed{\frac{2}{r}} \partial_r u$$

… but new algorithms handle singular terms and stabilize the numerics, *even when solving Einstein's equations*

(E.g., Baumgarte et al's 3+1 BSSN in Spherical Polar Coords, PhysRevD.87.044026)

New Goals for Numerical Relativity

- Handle arbitrary, *dynamical* coordinate systems
  - *Even those with coordinate singularities*
  - 200—2,000x speed-up, supercomputer→desktop!

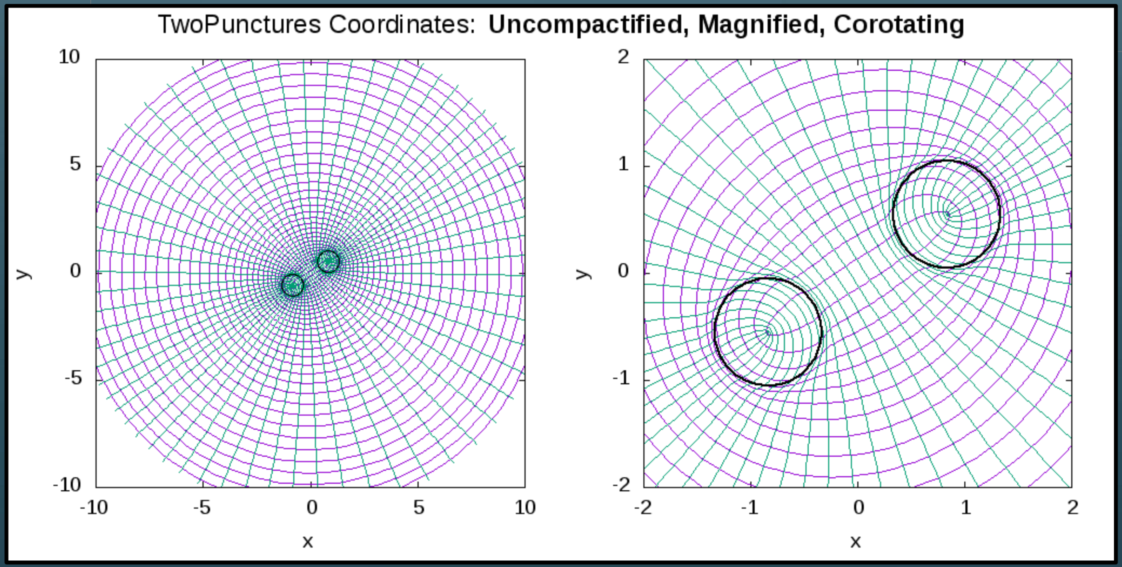


SENR: A Super-Efficient Numerical Relativity Code for the Age of Gravitational Wave Astrophysics

Zachariah B. Etienne
Ian Ruchlin

in collaboration with

Thomas W. Baumgarte



BOWDOIN

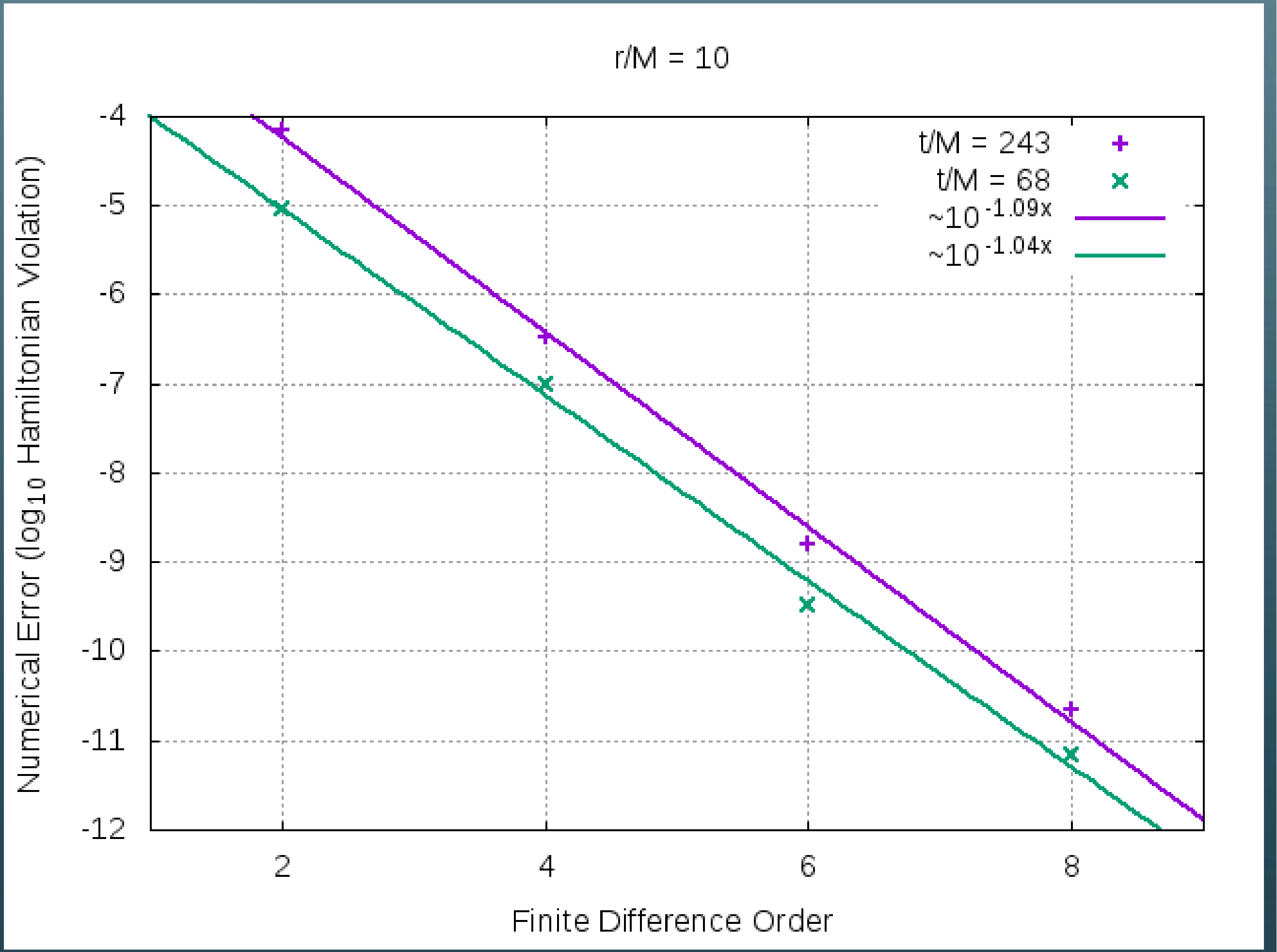
SENR Design Philosophy

- Open Source, Open Development → Greater Adoption
  - http://tinyurl.com/senrcode
- Algorithmic Simplicity → More Science Faster
  - Easier to debug & extend
  - Build on tried & true algorithms
    - BSSN in Spherical Polar Coords techniques pioneered by T. Baumgarte et al
      - SENR: Extend ideas to support arbitrary, *dynamical* coords
- Memory Efficiency Is Key Focus: Unlock the Desktop
  - Get public involved → ~10,000x more GW throughput!
- Bottom line: Maximize science with minimal human & computational resources

SENR Results: Convergence to exact solution, *even for black holes!*

Simulating black hole without excision:

Numerical errors converge to zero *exponentially* with increased polynomial approximation order!



Summary

- Electromagnetic counterparts to gravitational wave observations are likely!
- Enormous improvements will be necessary for numerical relativity to maximize the science from such observations
- The GiRaFFE & SENR (Super Efficient Numerical Relativity) codes aim to be big steps in this direction
- Stay tuned on our progress:

http://tinyurl.com/senrcode