A Possibility of Quark Spin Polarized Phase in High Density Quark Matter

Yasuhiko TSUE (Kochi Univ., Japan) with
J. da Providência (Univ. de Coimbra, Portugal)
C. Providência (Univ. de Coimbra, Portugal)
M. Yamamura (Kansai Univ., Japan)
H.Bohr (Danish Technical Univ., Denmark)

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Introduction

- Quark and/or hadronic matter at finite density
  - low density: hadronic phase : chiral broken phase
  - high density: quark–gluon or color superconducting phase
    : chiral symmetric phase

- Possibility of quark spin polarization?
  → Quark ferromagnetization?
  If possible, origin of spin polarization?

from nuclear matter or from quark matter?

Consider a possibility of spontaneous spin polarization in quark matter at high density

@K.Fukushima and T. Hatsuda
We show that the tensor-type four-point interaction between quarks leads to the NJL model with tensor interaction, even in the chiral symmetric phase (quark mass is zero), spontaneous spin polarization occurs at high density. This is further supported by the pseudovector interaction (cf. E. Nakano, T. Maruyama and T. Tatsumi, PRD 68 (2003) 105001) and the spin polarization disappears due to quarks being massless (cf. S. Maedan, PTP 118 (2007) 729).

Spin polarized phase survives against two-flavor color superconductivity and against color-flavor locking.
Model – NJL model

- Nambu–Jona–Lasinio model with tensor–type interaction

\[
\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_T \\
\mathcal{L}_{\text{kin}} = i \bar{\psi} \gamma^\mu \partial_\mu \psi \\
\mathcal{L}_S = -G_S [(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \bar{\tau} \psi)^2] \\
\mathcal{L}_V = -G_V [(\bar{\psi} \gamma^\mu \bar{\tau} \psi)^2 + (\bar{\psi} \gamma_5 \gamma^\mu \bar{\tau} \psi)^2] \\
\mathcal{L}_T = -G_T [(\bar{\psi} \gamma^\mu \gamma^\nu \bar{\tau} \psi)^2 + (\bar{\psi} i \gamma_5 \gamma^\mu \gamma^\nu \psi)^2]
\]

At high baryon density, chiral symmetry is restored

\[\langle \bar{\psi} \psi \rangle = 0 \quad : \quad \text{quarks are massless}\]

and then, \(\text{(S.Maeda, PTP 118 (2007) 729)}\)

\[\langle \bar{\psi} \gamma_5 \gamma^\mu \gamma^{\nu=3} \bar{\tau} \psi \rangle = 0 \quad : \quad \text{pseudovector condensate is zero} \]

\(\text{due to quark being massless}\)
Model – tensor interaction

- Tensor interaction is retained: \( L = L_{\text{kin}} + L_T \)

\[
\mathcal{L} = i\bar{\psi} \gamma^\mu \partial_\mu \psi \frac{G}{4} \left( \bar{\psi} \gamma^\mu \gamma^\nu \tau_3 \psi \right) \left( \bar{\psi} \gamma_\mu \gamma_\nu \tau_3 \psi \right)
\]

Here, \( \gamma^1 \gamma^2 = -i \Sigma_3 = -i \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \)

Then, \( \left\langle \bar{\psi} \gamma^1 \gamma^2 \tau_3 \psi \right\rangle \neq 0 \quad \rightarrow \quad \text{quark spin polarization occurs} \)

- Hereafter, \( \mu = 1, \nu = 2 \) are taken into account.
Effective potential – for spin polarization

- Generating functional $Z$

$$Z \propto \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp \left[ i \int d^4x \left( \bar{\psi}i\gamma^\mu \partial_\mu \psi + \frac{G}{2} (\bar{\psi}\Sigma^3\tau_k \psi)(\bar{\psi}\Sigma^3\tau_k \psi) \right) \right]$$

- Inserting “auxiliary field” $F_k \left( = -G\langle \bar{\psi}\Sigma^3\tau_k \psi \rangle \right)$

$$1 = \int \mathcal{D}F_k \exp \left[ -\frac{i}{2} \int d^4x \left( F_k + G(\bar{\psi}\Sigma^3\tau_k \psi) \right) G^{-1} \left( F_k + G(\bar{\psi}\Sigma^3\tau_k \psi) \right) \right]$$

Then, finally

$$Z \propto \int \mathcal{D}F_k \exp \left[ i \int d^4x \left( -\frac{F_k^2}{2G} + \frac{1}{4i} \text{tr} \ln \left( -p_0^2 + \epsilon_p^{(-)^2} \right) + \frac{1}{4i} \text{tr} \ln \left( -p_0^2 + \epsilon_p^{(+)^2} \right) \right) \right]$$

$$\epsilon_p^{(\pm)} = \sqrt{\left( (F_k\tau_k) \pm \sqrt{p_1^2 + p_2^2} \right)^2 + p_3^2} \quad \text{: single-particle energy}$$
Effective potential $-\text{ for spin polarization}$

- Effective potential: $V[F]$
  
  $$Z = \exp (i \Gamma[F]), \quad V[F] = -\frac{\Gamma[F]}{\int d^4x}$$

- At finite density, introduce chemical potential: $\mu$

  $$L \rightarrow L + \mu \overline{\psi} \gamma^0 \psi$$

  Then, finally

  $$V[F] = \frac{F^2}{2G} + 2N_c \int^F dF \int \frac{d^3p}{(2\pi)^3} \left[ \frac{F - \sqrt{p_1^2 + p_2^2}}{\epsilon_p(-)} \theta(\mu - \epsilon_p(-)) + \frac{F + \sqrt{p_1^2 + p_2^2}}{\epsilon_p(+) \theta(\mu - \epsilon_p(+) \right]$$

  where

  $$F_{k\tau} \rightarrow F_{\tau 3} = F_{\tau}, \quad \tau = \begin{cases} 
  1 & \text{for up quark} \\
  -1 & \text{for down quark}
  \end{cases}$$
Effective potential – for spin polarization

- Thermodynamic relation: pressure $p$
  - quark number density $\rho_q$

\[
p = -V[F], \quad \rho_q = -\frac{\partial V[F]}{\partial \mu}
\]
Effective potential: $V[F]$

Parameters used here:

$G = 20 \text{ GeV}^{-2}$

(with vacuum polarization, \( G = 11.1 \text{ GeV}^{-2} \) with cutoff \( \Lambda = 0.631 \text{ GeV} \))
Numerical results – for spin polarization

- Pressure vs chemical potential or baryon number density

- Critical density

<table>
<thead>
<tr>
<th>$G / \text{GeV}^{-2}$</th>
<th>$\rho_{cr} / \rho_0$</th>
<th>$\mu_{cr} / \text{GeV}$</th>
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</table>

($\rho_0 = 0.17 \text{ fm}^{-3}$)
Brief Summary

- We have shown • • •
  - tensor-type four-point interaction between quarks leads • • •
    - spontaneous quark spin polarization occurs at high density
Stability of spin polarized phase

--- two–flavor case

- High density (and low temperature) quark matter in two–flavor:
  - 2–flavor color superconducting (2SC) phase
  
  Is the spin polarized phase survives at high density against 2SC phase?
Interplay between spin polarization and color superconductivity

- Lagrangian density with 2-flavor color superconductivity

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_T + \mathcal{L}_c \]

\[ \mathcal{L}_c = \frac{G_c}{2} \sum_{A=2,5,7} \left( (\bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi^C)(\bar{\psi}^C i \gamma_5 \tau_2 \lambda_A \psi) + (\bar{\psi} \tau_2 \lambda_A \psi^C)(\bar{\psi}^C \tau_2 \lambda_A \psi) \right) \]

- Mean field approximation

\[ \mathcal{L}^{MF} = \mathcal{L}_0 + \mathcal{L}_T^{MF} + \mathcal{L}_c^{MF} \]

\[ \mathcal{L}_T^{MF} = -F(\bar{\psi} \Sigma_3 \tau_3 \psi) - \frac{F^2}{2G}, \quad (F = -G\langle \bar{\psi} \Sigma_3 \tau_3 \psi \rangle) \]

\[ \mathcal{L}_c^{MF} = -\frac{1}{2} \sum_{A=2,5,7} (\Delta \bar{\psi}^C i \gamma_5 \tau_2 \lambda_A + h.c.) - \frac{3\Delta^2}{2G_c}, \]

\[ \Delta_A = \Delta_A^* = -G_c \langle \bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi \rangle, \quad \Delta = \Delta_2 = \Delta_5 = \Delta_7 \]

For example, M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, PTP 108 (2002) 929
Interplay between spin polarization and color superconductivity

- **Hamiltonian formalism**

\[ \mathcal{H}_{MF} - \mu N = \mathcal{K}_0 + \mathcal{H}_{T}^{MF} + \mathcal{H}_{c}^{MF}, \]

\[ \mathcal{K}_0 = \bar{\psi}(-\gamma \cdot \nabla - \mu \gamma_0)\psi, \]

\[ \mathcal{H}_{T}^{MF} = -\mathcal{L}_{T}^{MF}, \quad \mathcal{H}_{c}^{MF} = -\mathcal{L}_{c}^{MF} \]

- **Hamiltonian for quark and antiquark**

\[ H_{MF} - \mu N = \sum_{p\eta \tau \alpha} [(p - \mu)c_{p\eta \tau \alpha}^{\dagger}c_{p\eta \tau \alpha} - (p + \mu)\tilde{c}_{p\eta \tau \alpha}^{\dagger}\tilde{c}_{p\eta \tau \alpha}] \]

\[ + \frac{F}{2} \sum_{p\eta \tau \alpha} \phi_{\tau} \left[ \frac{\sqrt{p_1^2 + p_2^2}}{p} \left( c_{p\eta \tau \alpha}^{\dagger}c_{p\eta \tau \alpha} - \eta \frac{p_3}{p} \right) \right] \]

\[ + \frac{\Delta}{2} \sum_{p\eta \alpha' \alpha'' \tau' \tau} \left( c_{p\eta \alpha}^{\dagger}c_{-p\eta \alpha'}^{\dagger} + \tilde{c}_{p\eta \alpha}^{\dagger}\tilde{c}_{-p\eta \alpha'}^{\dagger} + c_{-p\eta \alpha'}^{\dagger}c_{p\eta \alpha} + \tilde{c}_{-p\eta \alpha'}^{\dagger}\tilde{c}_{p\eta \alpha} \right) \phi_{\tau} \epsilon_{\alpha \alpha' \alpha'' \tau' \tau'} \]

\[ + V \cdot \frac{F^2}{2G} + V \cdot \frac{3\Delta^2}{2G_c} \]

where \( \eta = \pm 1 \) \( \cdots \) helicity, \( \tau = \pm 1 \) \( \cdots \) isospin \( (\phi_{\pm} = \pm 1), \quad \alpha \) \( \cdots \) color
Mean Field Approximation — for color-superconducting gap $\Delta$ and spin polarization $F$

Mean field approximation

\[
H_{\text{MF}} - \mu N = \sum_{p\eta\tau\alpha} \left[ (\epsilon_p^{(\eta)} - \mu) a_{p\eta\tau\alpha}^\dagger a_{p\eta\tau\alpha} - (\epsilon_p^{(\eta)} + \mu) \tilde{a}_{p\eta\tau\alpha}^\dagger \tilde{a}_{p\eta\tau\alpha} \right] \\
+ \frac{\Delta}{2} \sum f(\eta) \left[ a_{p\eta\tau\alpha}^\dagger a_{-p\eta\tau'\alpha''}^\dagger - \tilde{a}_{p\eta\tau\alpha}^\dagger \tilde{a}_{-p\eta\tau'\alpha''}^\dagger + h.c. \right] \epsilon_{\alpha'\alpha''} \epsilon_{\tau'\phi} \\
+ V \cdot \frac{F^2}{2G} + V \cdot \frac{3\Delta^2}{2G_c}.
\]

\[
\epsilon_p^{(\pm)} = \sqrt{p_3^2 + \left( F \pm \sqrt{p_1^2 + p_2^2} \right)^2}, \quad f(\eta) = \frac{p + \eta e}{\epsilon_p^{(\eta)}}, \quad \left( e = F \sqrt{p_1^2 + p_2^2} \right)
\]

\[
\Delta_A = \Delta_A^* = -G_c \langle \bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi \rangle, \quad \Delta = \Delta_2 = \Delta_5 = \Delta_7
\]

\[
F \tau_k = -G \langle \bar{\psi} \Sigma^3 \tau_k \psi \rangle
\]

$a_{p\eta\tau\alpha}$ ⋅ positive energy states, $\tilde{a}_{p\eta\tau\alpha}$ ⋅ negative energy states

Quark matter ⋅⋅⋅ positive energy particles are retained
Interplay between spin polarization and color superconductivity

**BCS state for positive energy particles**

\[ |\Psi\rangle = e^S |\Psi_0\rangle, \quad |\Psi_0\rangle = \prod_{p\eta\tau\alpha (\epsilon_p^{(\eta)} < \mu)} a_{p\eta\tau\alpha}^+ |0\rangle, \]

\[ S = \sum_{p\eta(\epsilon_p^{(\eta)} > \mu)} \frac{K_{p\eta}}{2} \sum_{a\alpha'a''\tau'} a_{p\eta\tau\alpha}^+ a_{-p\eta\tau'\alpha'}^+ \epsilon_{a\alpha'a''} \epsilon_{\tau\tau'} \phi_\tau + \sum_{p\eta(\epsilon_p^{(\eta)} \leq \mu)} \frac{\tilde{K}_{p\eta}}{2} \sum_{a\alpha'a''\tau'} a_{p\eta\tau\alpha}^+ a_{-p\eta\tau'\alpha'}^+ \epsilon_{a\alpha'a''} \epsilon_{\tau\tau'} \phi_\tau \]

where \( K_{p\eta} = K_{-p\eta}, \quad \tilde{K}_{p\eta} = \tilde{K}_{-p\eta} \)

and

\[ \sin \theta_{p\eta} = \frac{\sqrt{3}K_{p\eta}}{\sqrt{1+3K_{p\eta}^2}}, \quad \sin \tilde{\theta}_{p\eta} = \frac{\sqrt{3}\tilde{K}_{p\eta}}{\sqrt{1+3\tilde{K}_{p\eta}^2}} \]
Interplay between spin polarization and color superconductivity

- Variational equations determine $\theta(K)$

$$\frac{\partial}{\partial \theta_{p\eta}} \langle \Psi | H_{MF} - \mu N | \Psi \rangle = 0, \quad \frac{\partial}{\partial \tilde{\theta}_{p\eta}} \langle \Psi | H_{MF} - \mu N | \Psi \rangle = 0$$

- Thermodynamic potential

$$\Phi(\Delta, F, \mu) = \frac{1}{V} \langle \Phi | H_{MF} - \mu N | \Phi \rangle = 2 \cdot \frac{1}{V} \sum_{p\eta (\varepsilon_p^{(\eta)} \leq \mu)} \left[ 2 (\varepsilon_p^{(\eta)} - \mu) - \sqrt{ (\varepsilon_p^{(\eta)} - \mu)^2 + 3 \Delta^2 f(\eta)^2 } \right]$$

$$+ 2 \cdot \frac{1}{V} \sum_{p\eta (\varepsilon_p^{(\eta)} > \mu)} \left[ (\varepsilon_p^{(\eta)} - \mu) - \sqrt{ (\varepsilon_p^{(\eta)} - \mu)^2 + 3 \Delta^2 f(\eta)^2 } \right] + \frac{F^2}{2G} + \frac{3\Delta^2}{2G_c}$$
Interplay between spin polarization and color superconductivity

Gap equation

$$\frac{\partial}{\partial \Delta} \langle \Phi \mid H_{MF} - \mu N \mid \Phi \rangle = 0$$

Namely,

$$\Delta \left[ 2 \cdot \frac{1}{V} \sum_{\eta=\pm}^{\Lambda} \frac{f(\eta)^2}{\sqrt{(\epsilon_p^{(\eta)} - \mu)^2 + 3\Delta^2 f(\eta)^2}} - \frac{1}{G_c} \right] = 0$$
Numerical results

- for interplay between spin polarization and 2SC

Occupation number for 2SC phase

Usual cutoff $\Lambda = 0.631$ GeV is valid for this calculation

<table>
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Numerical results

- for interplay between spin polarization and 2SC

- Thermodynamic potential

![Graphs showing thermodynamic potential for different values of chemical potential (mu)]
We have shown •••

- tensor-type four-point interaction between quarks leads •••
  - spontaneous spin polarization occurs at high density even in chiral symmetric phase (quark mass is zero),

- spin polarized (SP) phase survives against two-flavor color superconducting (2SC) phase at high density

- the order of phase transition from 2SC to SP may be the second order
Stability of spin polarized phase

---- three-flavor case

- High density (and low temperature) quark matter in three-flavor:
  - color-flavor locked (CFL) phase may exist

Is the spin polarized phase survives at high density against CFL phase?
Interplay between spin polarization and color superconductivity

- Lagrangian density with 3-flavor color superconductivity

\[
L = \bar{\psi} i \gamma^\mu \partial_\mu \psi - \frac{G}{4} \left( \bar{\psi} \gamma^\mu \gamma^\nu \lambda^f_k \psi \right) \left( \bar{\psi} \gamma_\mu \gamma_\nu \lambda^f_k \psi \right) + \frac{G_c}{2} \left( \bar{\psi} i \gamma_5 \lambda_c \lambda^f_k \psi \right) \left( \bar{\psi}^c i \gamma_5 \lambda^c_a \lambda^f_k \psi \right)
\]

- Mean field approximation

\[
L = \bar{\psi} i \gamma^\mu \partial_\mu \psi + L^{MF}_T + L^{MF}_c
\]

\[
L^{MF}_T = - \sum_{k=3,8} F_k \left( \bar{\psi} \Sigma_3 \lambda^f_k \psi \right) - \frac{1}{2G} \sum_{k=3,8} F_k^2
\]

\[
\Sigma_3 = -i \gamma^1 \gamma^2 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad F_3 = -G \left( \bar{\psi} \Sigma_3 \lambda^f_3 \psi \right), \quad F_8 = -G \left( \bar{\psi} \Sigma_3 \lambda^f_8 \psi \right)
\]

\[
L^{MF}_c = - \frac{1}{2} \sum_{(a,k) \in \{2,5,7\}} \left( \Delta^*_{ak} \left( \bar{\psi}^c i \gamma_5 \lambda^c_a \lambda^f_k \psi \right) + h.c. \right) + \frac{1}{2G_c} \left| \Delta_{ak} \right|^2
\]

\[
\Delta_{ak} = -G_c \left( \bar{\psi} \psi^c i \gamma_5 \lambda^c_a \lambda^f_k \psi \right)
\]
Interplay between spin polarization and color superconductivity

- **Hamiltonian formalism**

\[ \mathcal{H}_{MF} - \mu N = \mathcal{K}_0 + \mathcal{H}_{TM}^{MF} + \mathcal{H}_{c}^{MF}, \]
\[ \mathcal{K}_0 = \bar{\psi}(-\gamma \cdot \nabla - \mu \gamma_0)\psi, \]
\[ \mathcal{H}_{TM}^{MF} = -\mathcal{L}_{TM}^{MF}, \quad \mathcal{H}_{c}^{MF} = -\mathcal{L}_{c}^{MF} \]

- **Hamiltonian for quark and antiquark**

\[ H = H_0 - \mu N + V_{CFL} + V_{SP} + V \cdot \frac{1}{2G} \left( F_3^2 + F_8^2 \right) + V \cdot \frac{3\Delta^2}{2G_c} \]
\[ H_0 - \mu N = \sum_{p \eta \tau \alpha} \left[ |p| - \mu \right] c_{p \eta \tau \alpha}^+ c_{p \eta \tau \alpha} - \left[ |p| + \mu \right] \bar{c}_{p \eta \tau \alpha}^+ \bar{c}_{p \eta \tau \alpha} \]

\[ V_{CFL} = \frac{\Delta}{2} \sum_{p \eta \alpha \alpha'} \sum_{\tau \tau'} \left( c_{p \eta \tau \alpha}^+ c_{-p \eta \tau' \alpha'} + c_{-p \eta \tau' \alpha} c_{p \eta \tau \alpha} + \bar{c}_{p \eta \tau \alpha}^+ \bar{c}_{-p \eta \tau' \alpha'} + \bar{c}_{-p \eta \tau' \alpha} \bar{c}_{p \eta \tau \alpha} \right) \epsilon_{\alpha \alpha' \tau \tau'} \phi_p \]

\[ V_{SP} = \sum_{p \eta \tau \alpha} F_{\tau} \left[ \sqrt{p_1^2 + p_2^2} \right] \left( c_{p \eta \tau \alpha}^+ c_{-p \eta \tau \alpha} + \bar{c}_{p \eta \tau \alpha}^+ \bar{c}_{-p \eta \tau \alpha} \right) - \eta \frac{p_3}{|p|} \left( c_{p \eta \tau \alpha}^+ \bar{c}_{p \eta \tau \alpha} + \bar{c}_{p \eta \tau \alpha}^+ c_{p \eta \tau \alpha} \right) \]

where

\[ \eta = \pm 1 \cdots \text{helicity}, \quad \tau = u, d, s \cdots \text{flavor}, \quad \alpha \cdots \text{color} \quad (\phi_p = -\phi_{\bar{p}} = 1) \]
Mean Field Approximation – for color–superconducting gap $\Delta$ without spin polarization $F (=0)$

Mean field approximation for quasi-particle operators

$$ H_{CFL} = H_0 - \mu N + V_{CFL} + V \cdot \frac{3\Delta^2}{2G_c} $$

$$ = \frac{1}{2} \sum_{|p|>\mu} \left[ 9\bar{\epsilon}_p - \sqrt{\bar{\epsilon}_p^2 + 4\Delta^2} - 8\sqrt{\bar{\epsilon}_p^2 + \Delta^2} \right] + \sum_{|p|<\mu} \left[ \sqrt{\bar{\epsilon}_p^2 + 4\Delta^2} d_{p;1}^+ d_{p;1} + \frac{9}{a=2} \sqrt{\bar{\epsilon}_p^2 + \Delta^2} d_{p,a}^+ d_{p,a} \right] $$

Thermodynamic potential for $F=0$

$$ \Phi_0 = \frac{1}{V} \langle H_{CFL} \rangle $$

$$ = \frac{1}{2V} \sum_{|p|>\mu} \left[ 9\bar{\epsilon}_p - \sqrt{\bar{\epsilon}_p^2 + 4\Delta^2} - 8\sqrt{\bar{\epsilon}_p^2 + \Delta^2} \right] + \frac{1}{2V} \sum_{|p|<\mu} \left[ 9\bar{\epsilon}_p - \sqrt{\bar{\epsilon}_p^2 + 4\Delta^2} - 8\sqrt{\bar{\epsilon}_p^2 + \Delta^2} \right] + \frac{3\Delta^2}{2G_c} $$
Mean Field Approximation — for spin polarized gap $F$ without CFL condensate $\Delta (=0)$

Thermodynamic potential for $\Delta=0$

$$
\Phi_F = 3 \cdot \frac{1}{V} \sum_{p, \eta=\pm, \tau=u,d,s} \left( \varepsilon_{p\tau}^{(\eta)} - \mu \right) \theta(\mu - \varepsilon_{p\tau}^{(\eta)}) + \frac{1}{2G} \left( F_3^2 + F_8^2 \right)
$$

$$
\varepsilon_{p\tau}^{(\eta)} = \sqrt{p_3^2 + \left( F_\tau + \eta \sqrt{p_1^2 + p_2^2} \right)^2}, \quad F_\tau = \left( F_3 + \frac{1}{\sqrt{3}} F_8 \right) \delta_{\tau u} + \left( -F_3 + \frac{1}{\sqrt{3}} F_8 \right) \delta_{\tau d} - \frac{2}{\sqrt{3}} F_8 \delta_{\tau s}
$$

Gap equations for $\Phi_0$ (CFL), $\Phi_F$ (SP)

$$
\frac{\partial \Phi_0}{\partial \Delta} = 0, \quad \frac{\partial \Phi_F}{\partial F_3} = \frac{\partial \Phi_F}{\partial F_8} = 0
$$
Numerical results

- for interplay between spin polarization and CFL

- Pressure $p$ vs chemical potential $\mu$

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Order of phase transition

- second order perturbation on CFL phase with respect to SP term

- Hamiltonian under consideration

\[ H = H_{CFL} + H_{SP}, \quad H_{SP} = \sum_{p \eta \alpha} F_{\tau} \sqrt{p_1^2 + p_2^2} \ \frac{c^+_{p \eta \alpha} c_{p - \eta \alpha}}{|p|} \]

Here, \( H_{SP} \) is regarded as perturbation term

- First order perturbation = 0

- Second order perturbation

\[ E_{\text{corr}} = \sum_i \frac{\langle \Phi | H_1 | i \rangle \langle i | H_1 | \Phi \rangle}{E_0 - E_i} \]

\( E_0 \); ground state energy, \( |i\rangle \); intermediate (excited) state, \( E_i \); excited state energy
Order of phase transition

- second order perturbation on CFL phase with respect to SP term

- Thermodynamic potential

\[ \Phi = \Phi_0 + \frac{1}{V} E_{\text{corr}} + \frac{1}{2G} \left( F_3^2 + F_8^2 \right) \]

\[ \Phi = \Phi_0 + \left( c_3 + \frac{1}{2G} \right) F_3^2 + \left( c_8 + \frac{1}{2G} \right) F_8^2 \]

<table>
<thead>
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<th>( \mu / \text{GeV} )</th>
<th>( c_3 + 1/(2G) )</th>
<th>( c_8 + 1/(2G) )</th>
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<td>0.012367</td>
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</table>

coefficients of \( F_3 \) and \( F_8 \) are always positive

\[ \Delta \neq 0, \quad F_3 = F_8 = 0 \text{ is local minimum} \]
Thus, $\Delta \neq 0$ and $F_3 = F_8 = 0$ — stable

Then, the phase transition may be the first order
We have shown ⋄ ⋄

- tensor-type four-point interaction between quarks leads ⋄ ⋄
  - spontaneous spin polarization occurs at high density even in chiral symmetric phase (quark mass is zero),

- spin polarized (SP) phase survives against color–flavor locking (CFL) phase at high density

- the order of phase transition from CFL to SP may be the first order