Numerical Simulations of Black Hole Accretion

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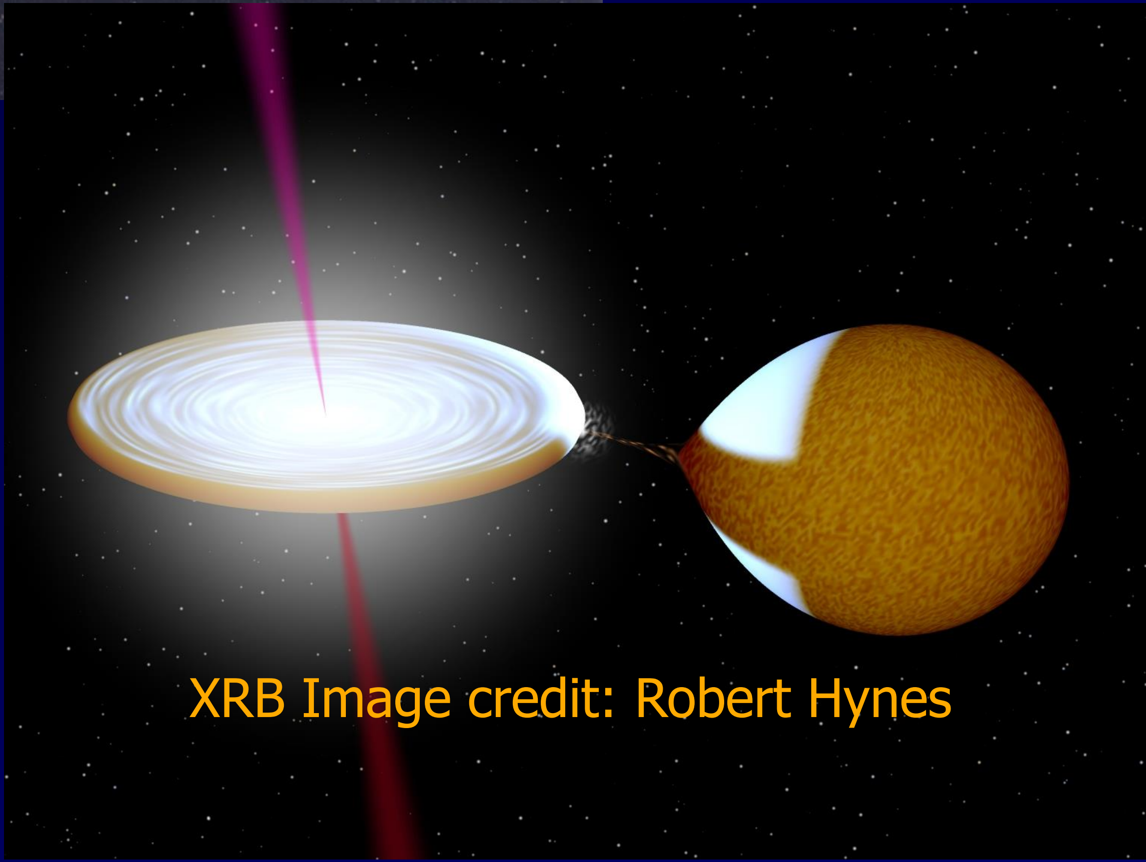
Black Hole Accretion Disks

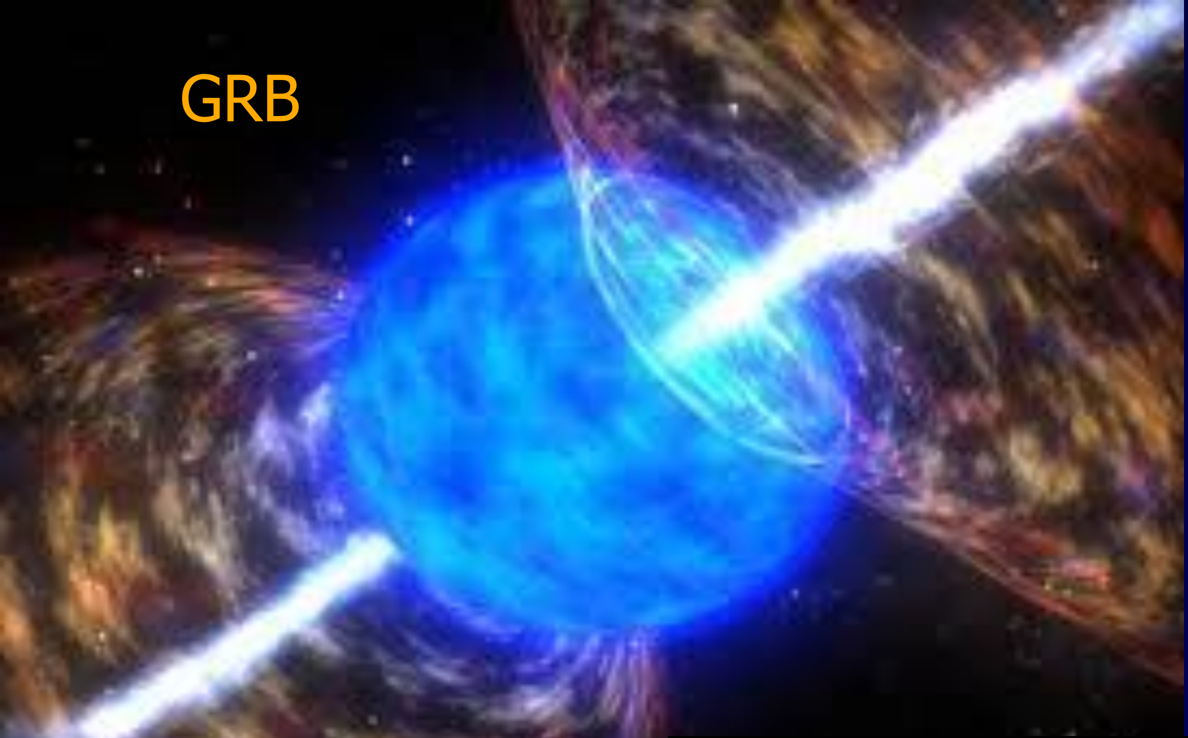
- Many kinds of accreting BHs in the universe
  - Active galactic nuclei: AGN
  - X-ray binaries: XRB
  - Gamma-ray bursts: GRB
  - Tidal disruption events: TDE

- Surprisingly diverse phenomena
  - Huge amount of data: spectra, variability…
  - Relativistic jets
  - Energy/mmtm feedback from AGN: M_{BH}-σ_{bulge}
  - **There are different accretion regimes**

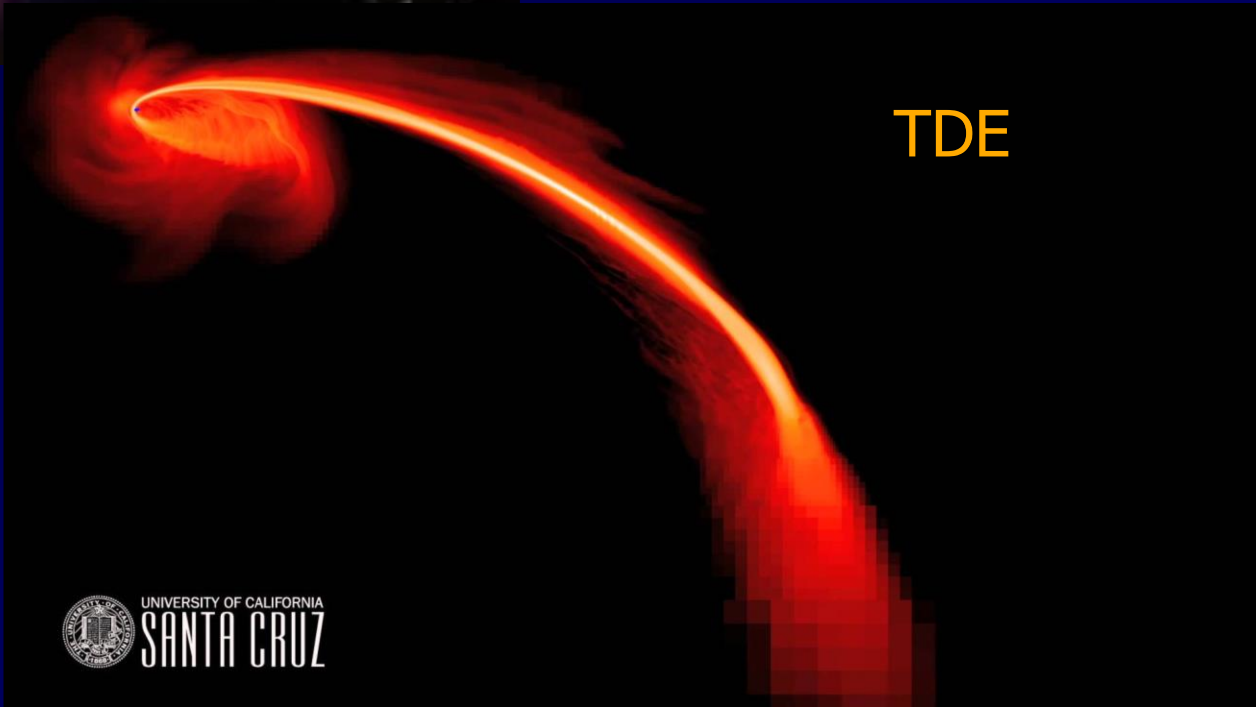
AGN Image credit: M. Inoue



XRB Image credit: Robert Hynes

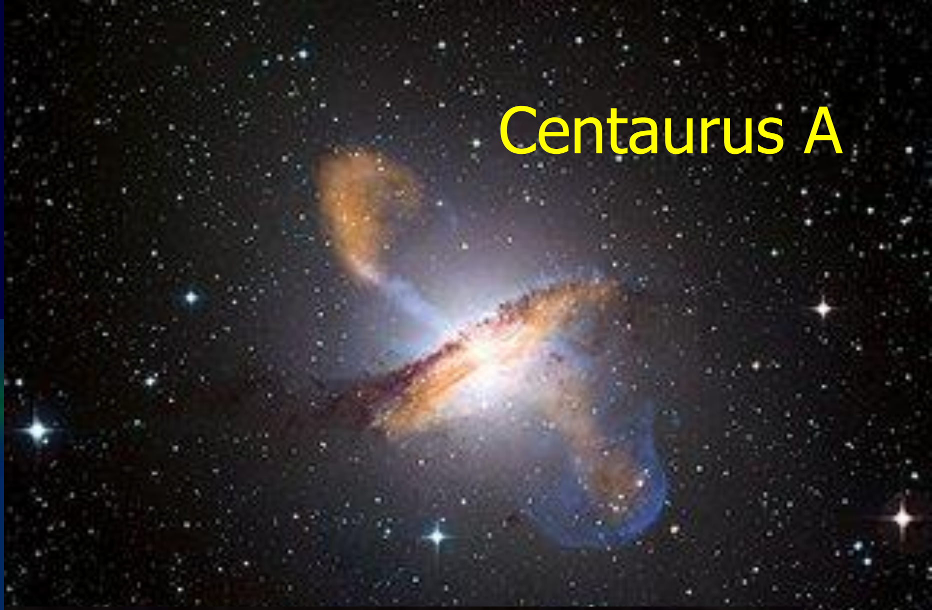


GRB

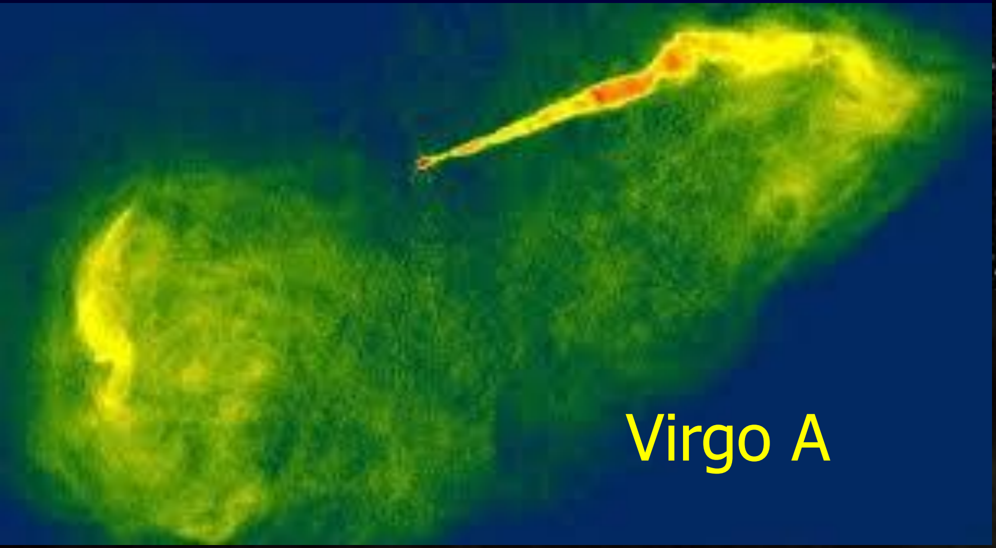


TDE

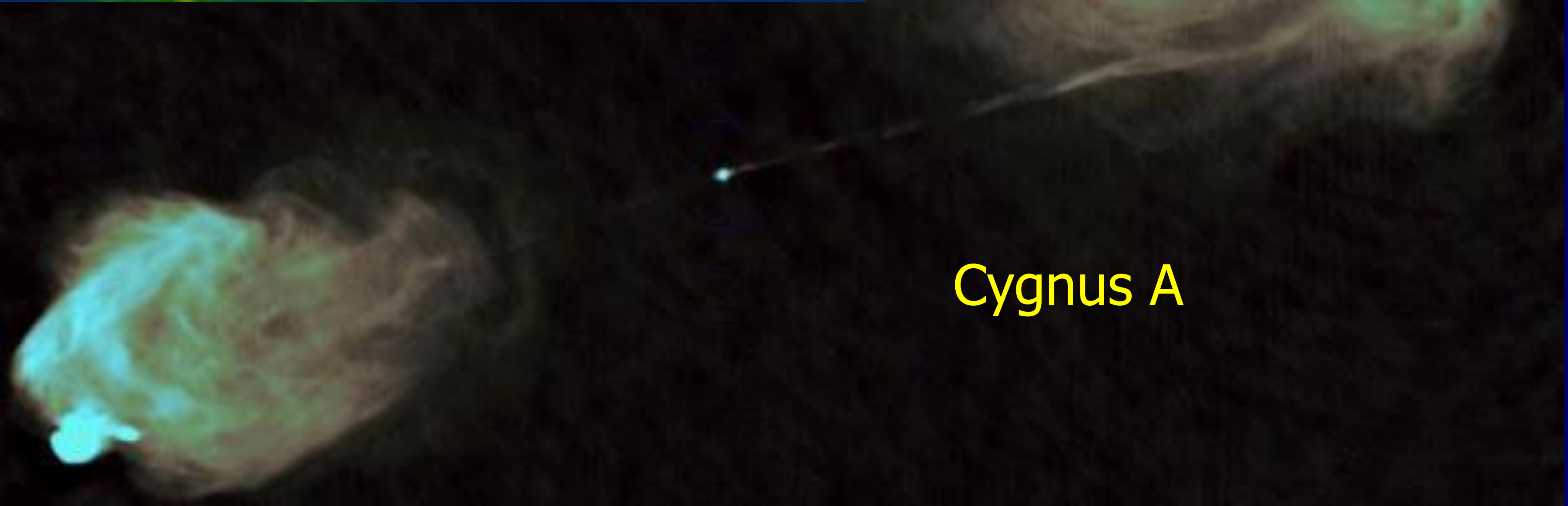
Relativistic Jets



Centaurus A



Virgo A

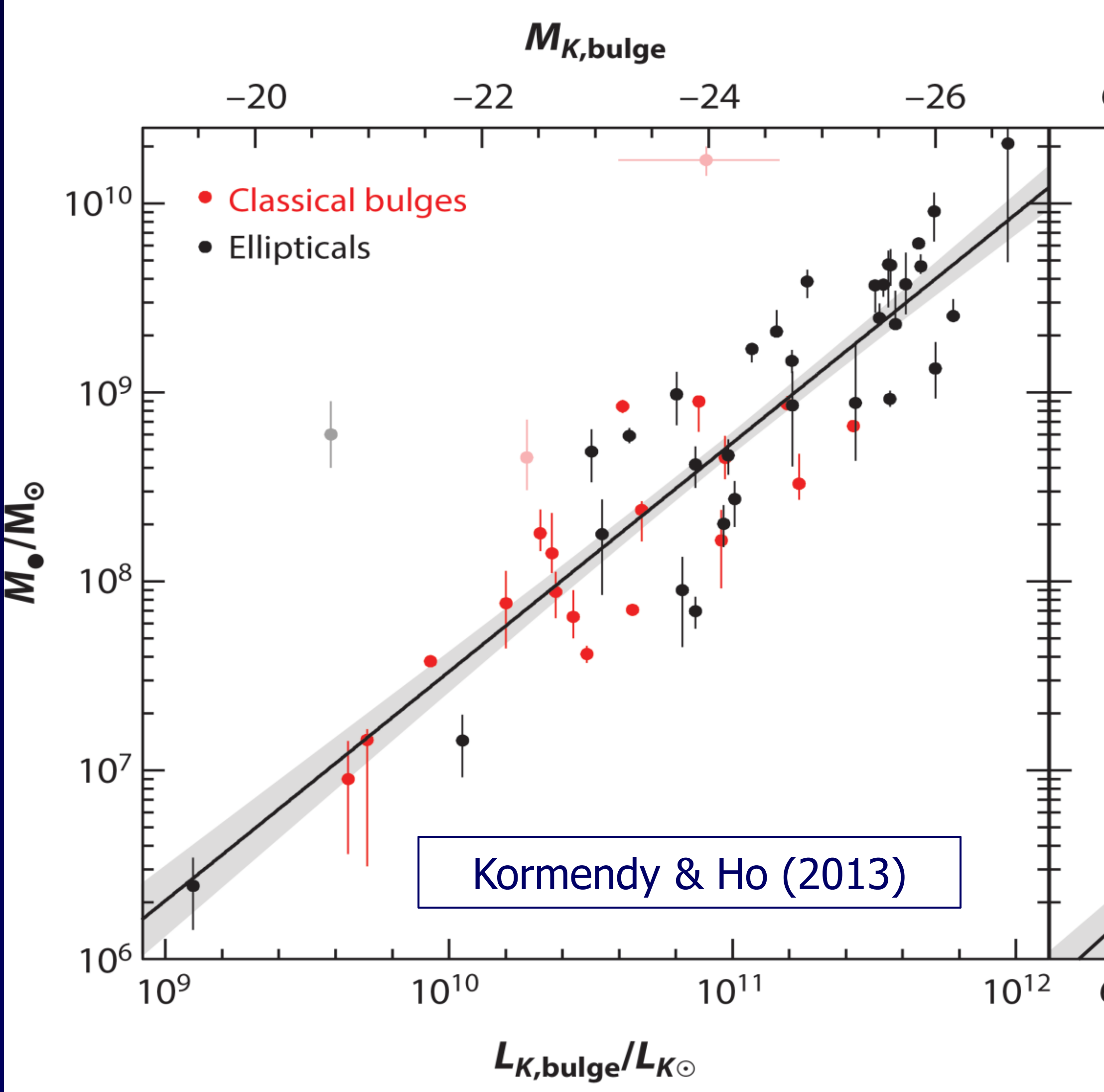


Cygnus A

The BH-Bulge Relation

There is a remarkable correlation between the mass M of the central supermassive black hole and the luminosity L of the host galaxy

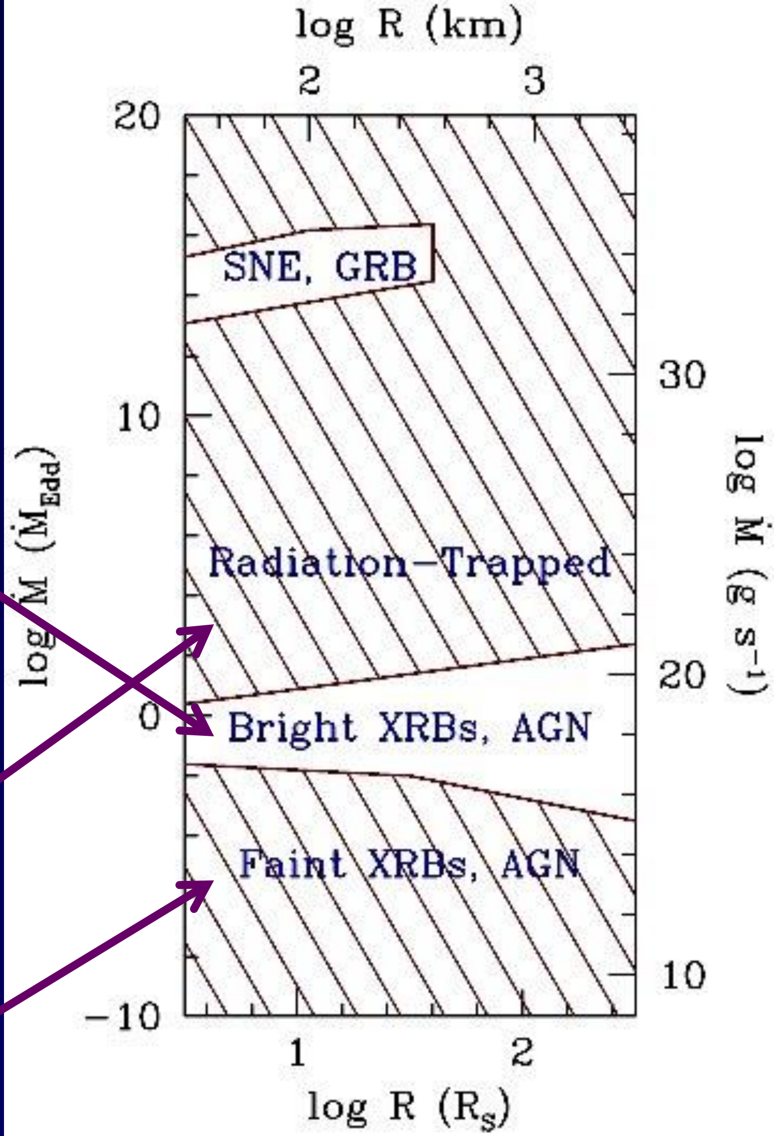
Important clue on the formation/evolution of SMBHs and galaxies



Kormendy & Ho (2013)

Accretion *Regimes:*

- **Thin Accretion Disk**:
  - Bright QSOs
  - XRBs in the Thermal State
- **Advection Dominated Accretion Flow (ADAF)**:
  - Radiation-trapped ADAF (Slim Disk)
  - Hot radiatively inefficient ADAF (RIAF)



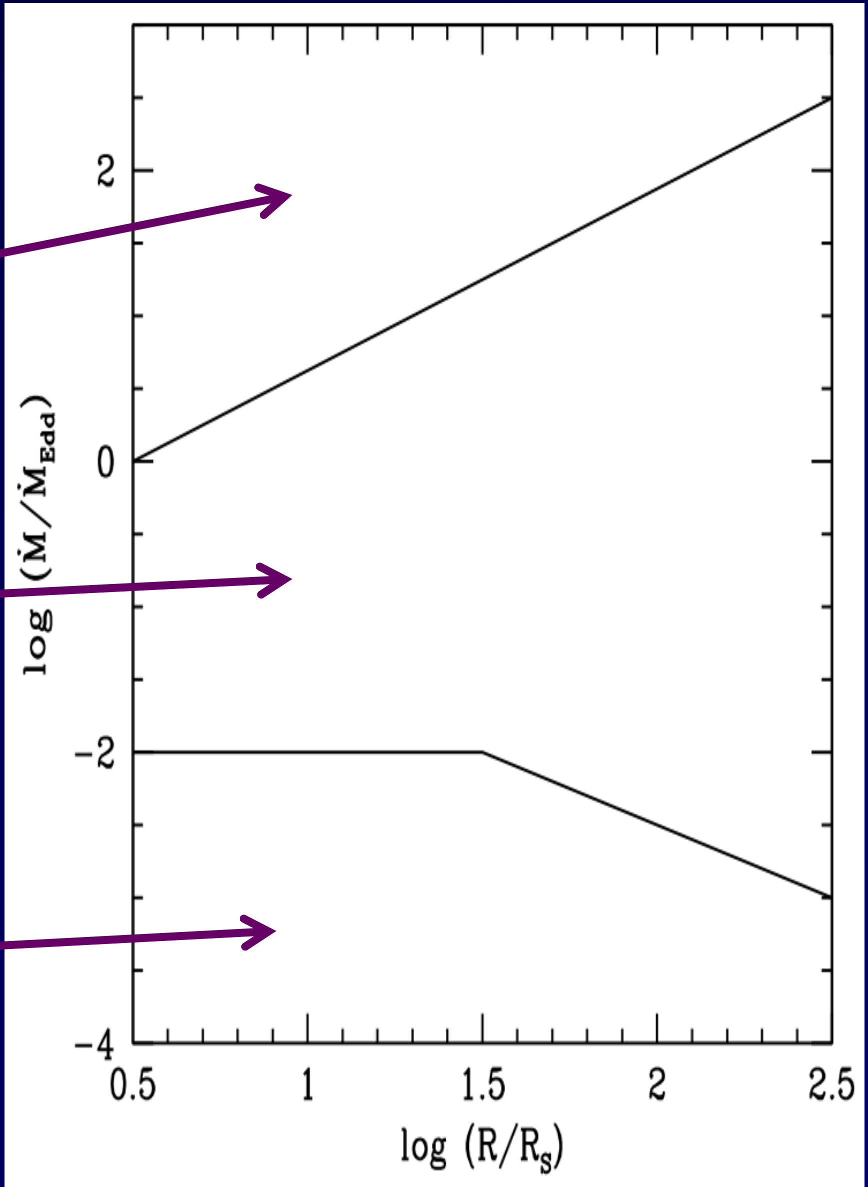
Narayan & Quataert (2005)
(M = 3M)

Accretion Regimes

Hyper-accretion, slim disk, ADAF
(Abramowicz et al. 1989; N & Yi 94)
Super-Eddington accretion
TDEs, ULXs, SS433

Thin accretion disk: radiatively efficient
(Shakura-Sunyaev, Novikov-Thorne 73)
Typical QSOs, Seyferts
XRBs in thermal soft state

Hot Accretion, ADAF, radiatively inefficient (Narayan & Yi 94, 95; Abramowicz et al. 95; Yuan & N 2014)
LLAGN, BL Lac objects, Sgr A*, M87
XRBs in hard state, quiescent state



Analytical Disk Models

- Useful 1D models have been derived for all three regimes by simplifying the equations and integrating vertically
- These solutions provide a lot of insight
- However, vertically integrated 1D models cannot describe jets and winds
- These phenomena are inherently 2D
- Need numerical simulations

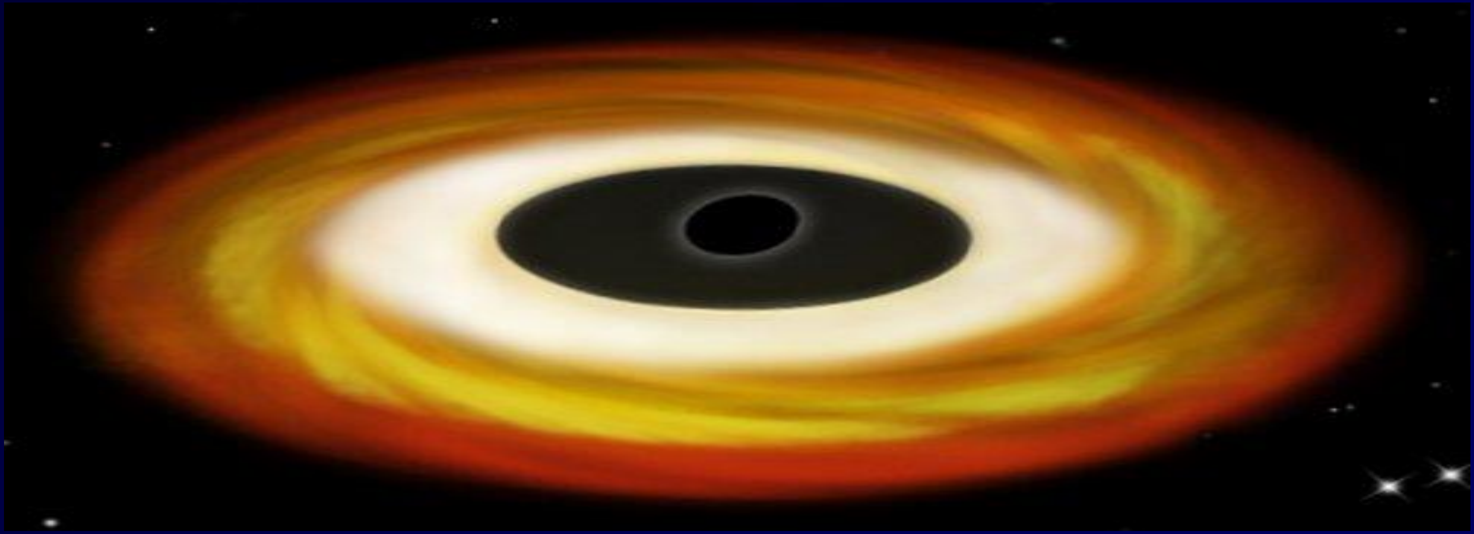
Numerical Simulations

- Numerical simulations can include all the complex physics that purely analytical methods cannot handle
  - Magnetic fields (MRI – “viscosity”) → **MHD**
  - Multi-dimensional → **3D MHD** (for MRI)
  - General relativity (BH) → **3D GRMHD**
  - Radiation → **3D GRRMHD**

Brief History

- Hydrodynamics (HD): early years
- Local MHD "shearing sheet" (Hawley & Balbus 1991; Gammie, Stone: 1990s)
- Global MHD full disk models (Stone, Igumenshchev, Hawley: ~2000)
- GRMHD (Koide, Gammie, McKinney, Hawley, de Villiers: early 2000s)
- Global radiation MHD (Ohsuga 2000s)
- GRRMHD (Sadowski, McKinney, Fragile: 2014)

Accretion: The Angular Momentum Problem



- Accreting gas has angular momentum and goes into Keplerian orbit around the BH

$$\boxed{\Omega_K(r) = \left(GM/r^3\right)^{1/2}, \quad l_K(r) = \left(GMr\right)^{1/2}}$$

- Gas must lose angular momentum to accrete

Angular Momentum Transfer

- Differential rotation gives a natural shear flow
- But microscopic viscosity is negligibly small
- Also, there is no hydrodynamic instability (even though the Reynolds number is enormous)
- Magneto-Rotational Instability (MRI, Balbus & Hawley 1991) drives MHD turbulence and causes angular momentum transfer

Magneto_Rotational Instability (MRI)

MRI: Differentially-rotating flow with a weak vertical magnetic field is linearly unstable

(Balbus & Hawley 1991)

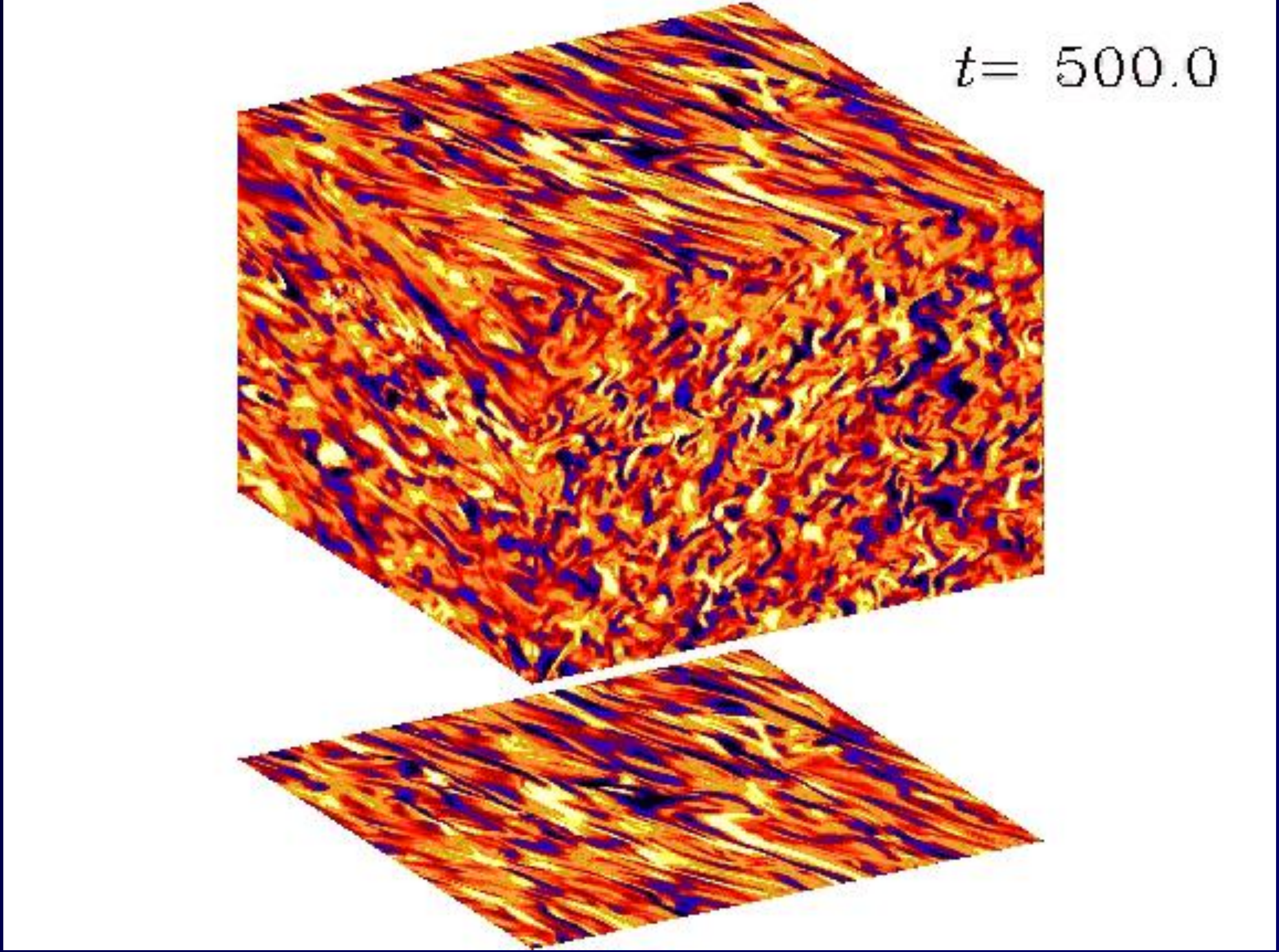
Clearly seen in local MHD simulations in a "shearing sheet"

Non-linear development of the MRI gives MHD turbulence, which transports angular mmtm (disk "viscosity")

φ

R

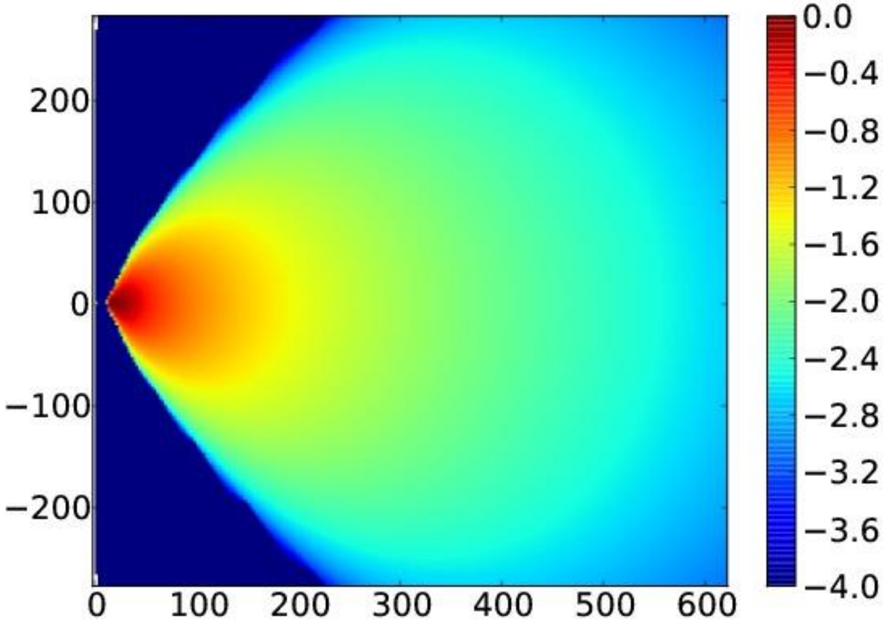
Movie courtesy: Charles Gammie



Movie courtesy: Axel Brandenberg

Global Disk Simulations

- Here, the entire disk (inner regions) is simulated in 3D/2D, usually in spherical coordinates
- Gas initially orbits in an axisymmetric torus with a weak magnetic field
- Once the simulation starts, the MRI grows and MHD turbulence causes gas to accrete
- With increasing time, the accretion flow reaches steady state over progressively larger radii, and disk properties can be studied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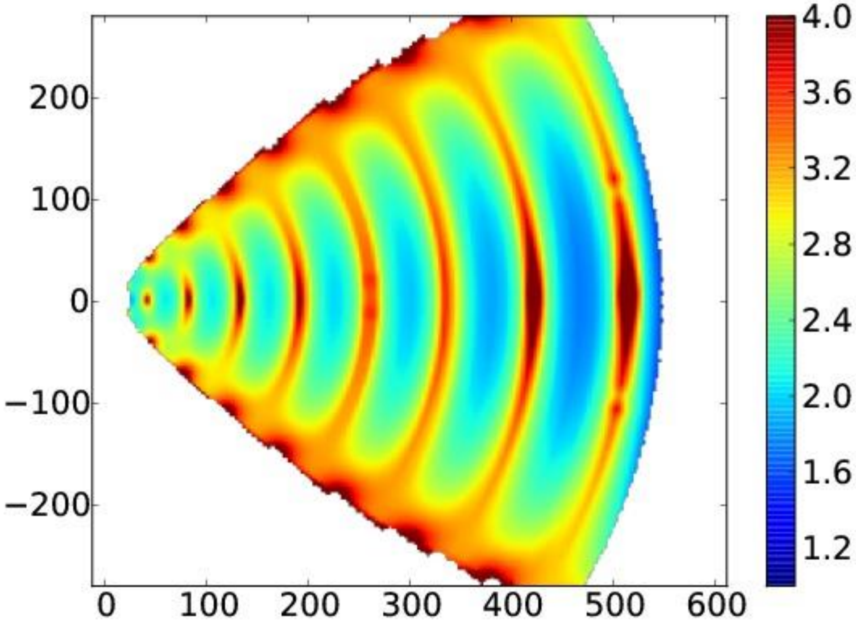
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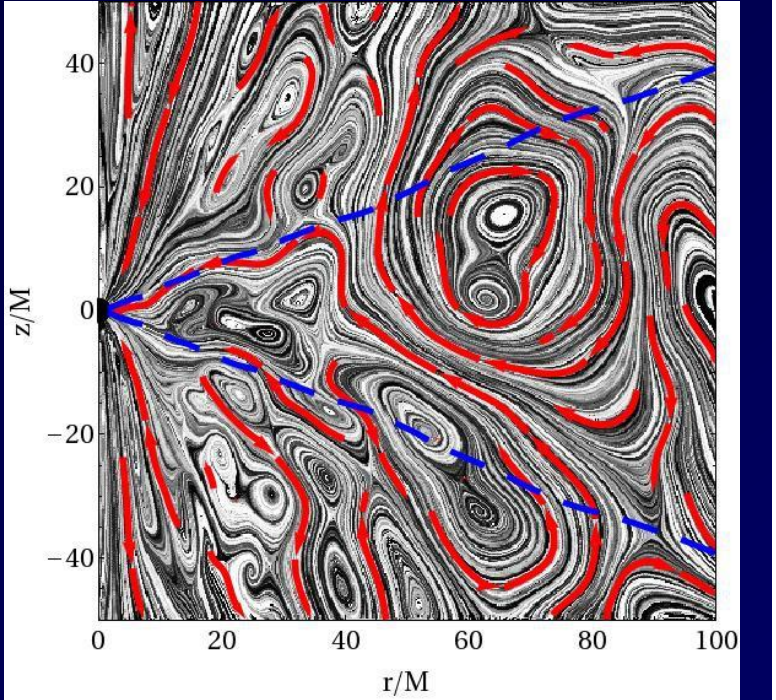
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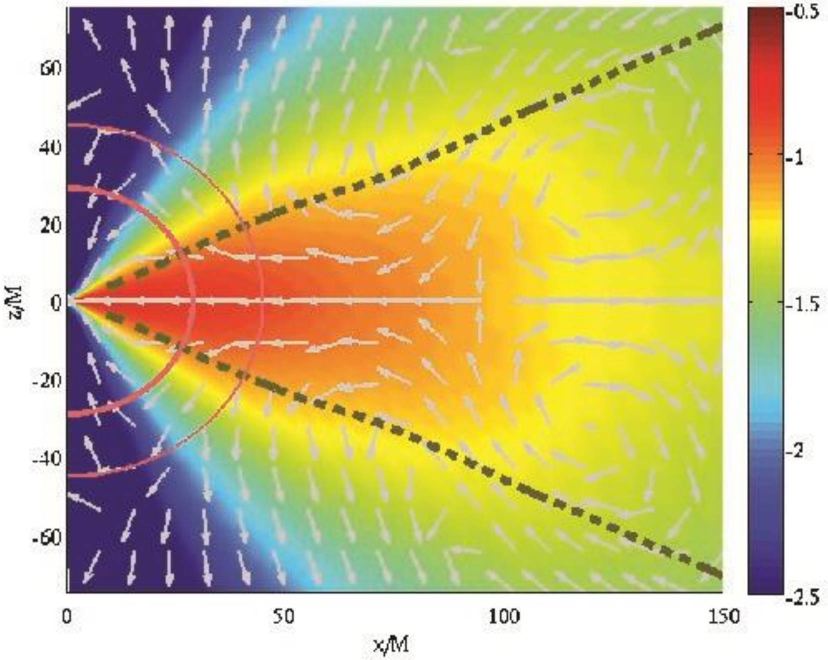
r/M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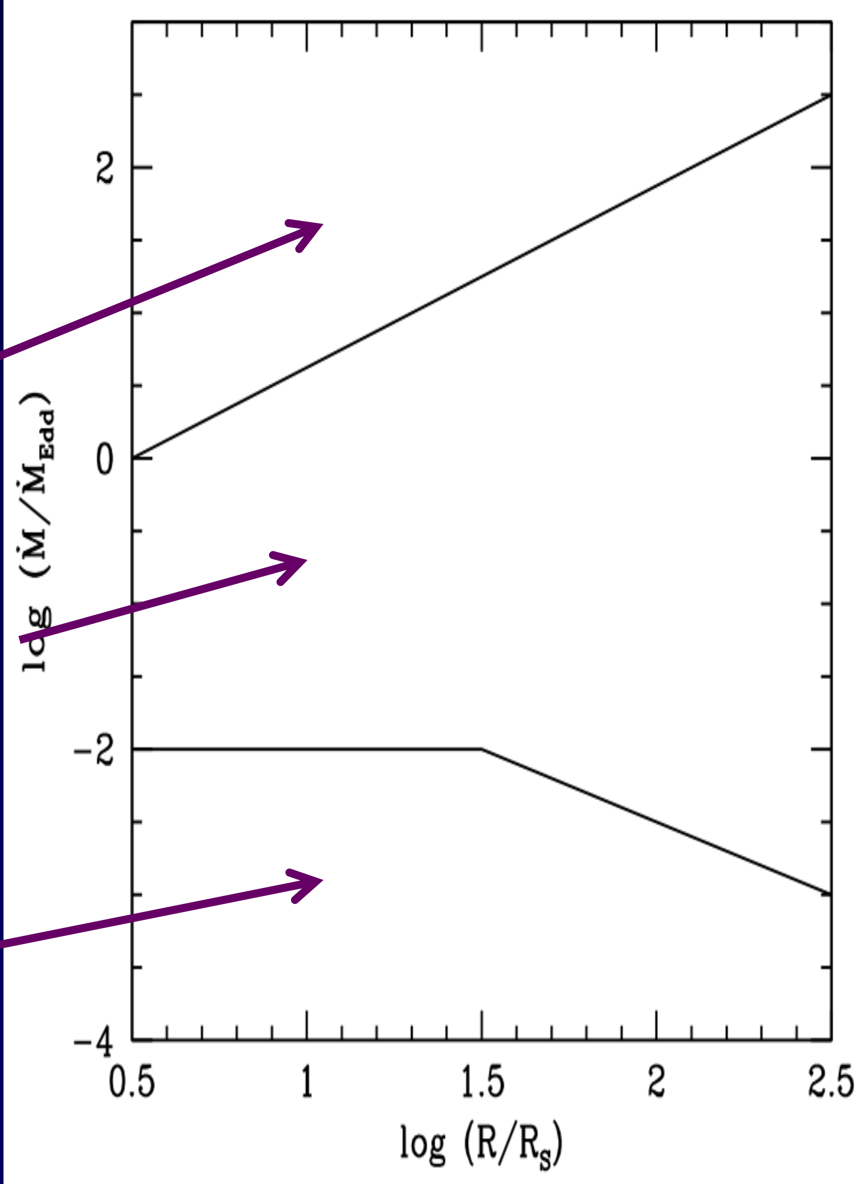
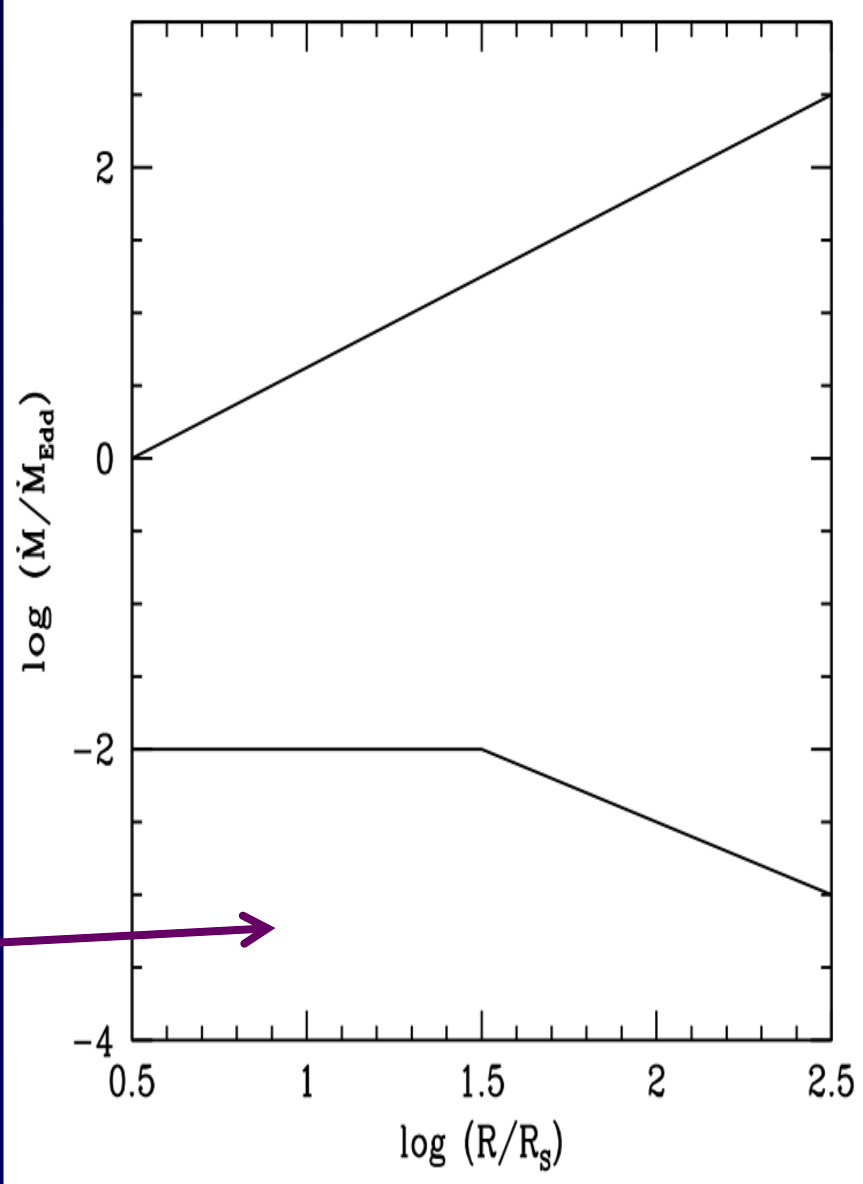
100
150
x/M
-0.5
-1
-1.5
-2
-2.5

Numerical Simulations of BH Accretion Disks

- Most difficult (radiation crucial)
- Intermediate difficulty (can use approximations to handle radiative cooling)
- Easiest to simulate (can neglect radiation)



Accretion Regimes

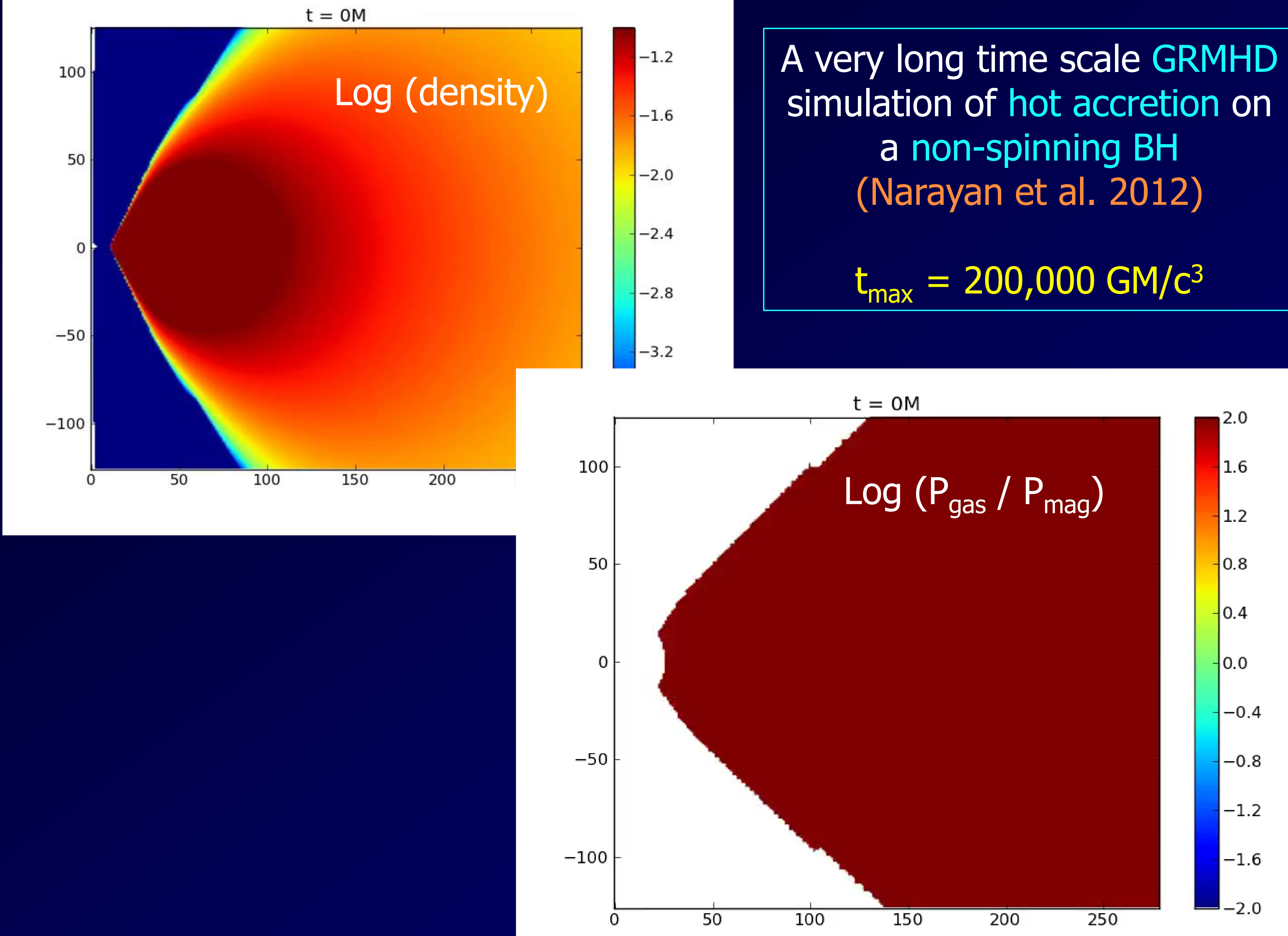


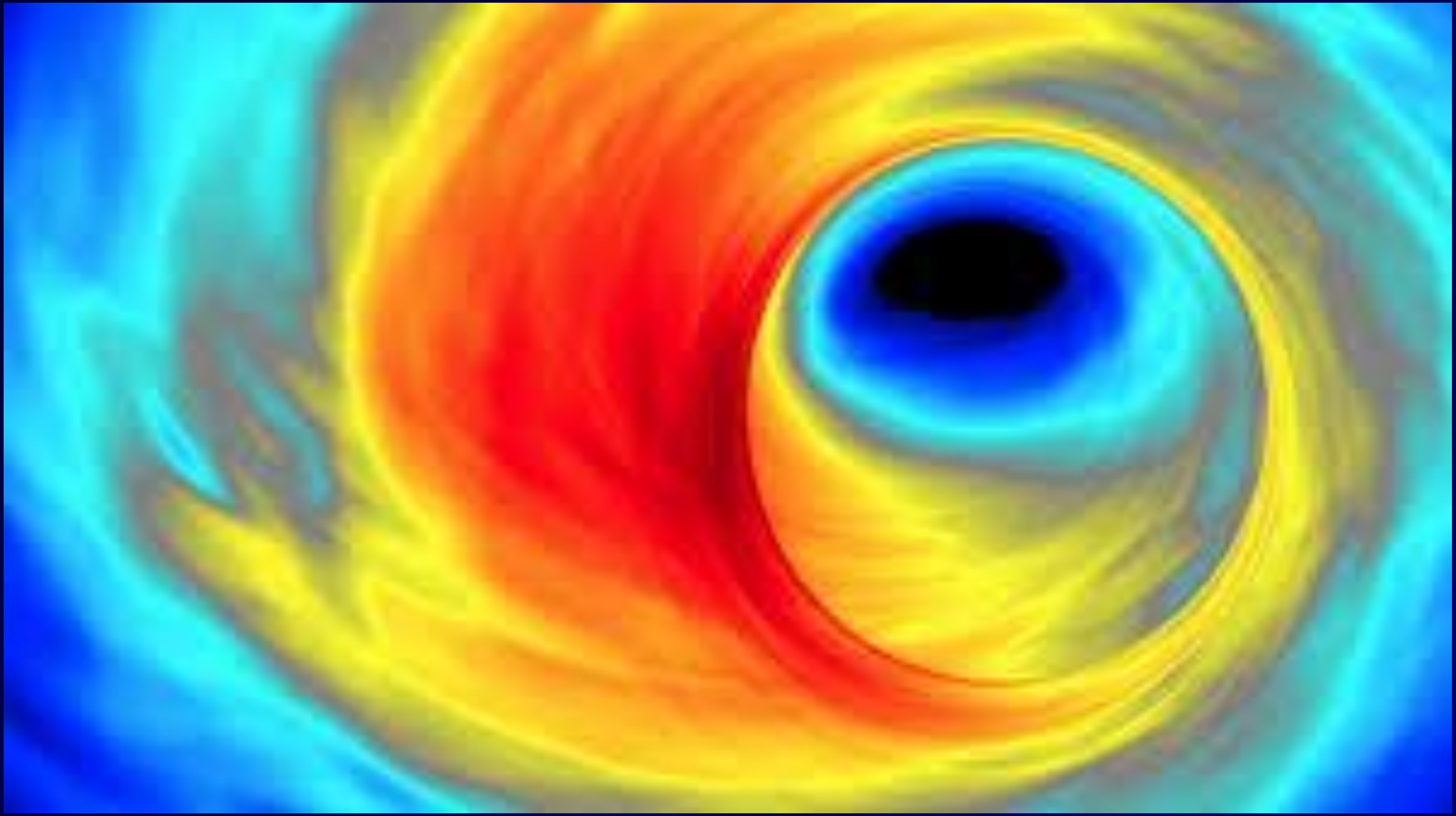
Hot Accretion, ADAF, radiatively inefficient (Narayan & Yi 94, 95; Abramowicz et al. 95; Yuan & N 2014)
LLAGN, BL Lac objects, Sgr A*, M87
XRBs in hard state, quiescent state

A very long time scale GRMHD simulation of hot accretion on a non-spinning BH

(Narayan et al. 2012)

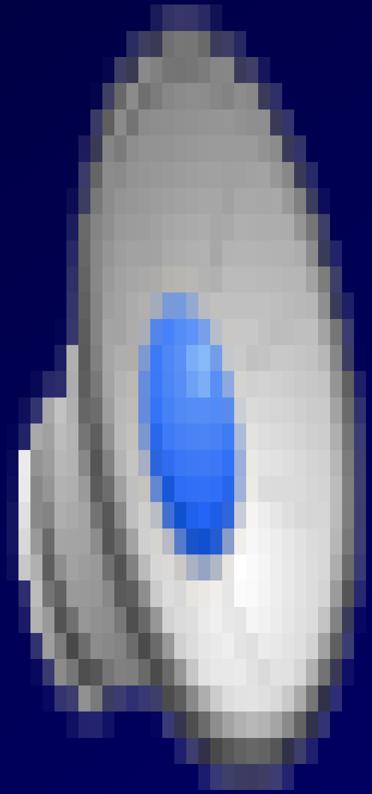
$t_{max} = 200{,}000\ GM/c^3$





Computer simulation image of gas accreting on the supermassive BH at the center of our Galaxy (Scott Noble)

Chi-Kwan Chan (2015)



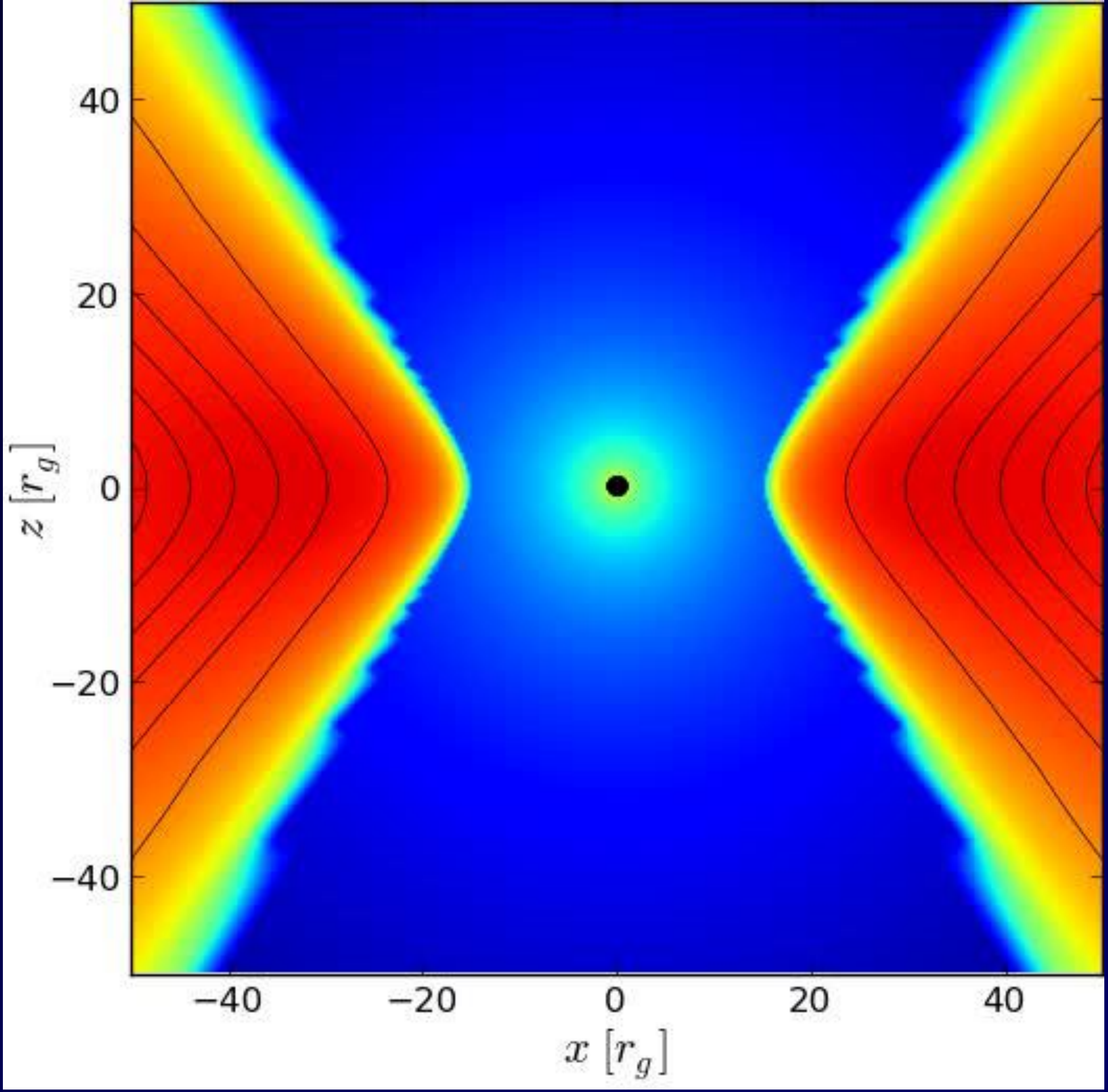
Radiation Post-Processing

- Simulations are good for studying the dynamics of the accreting gas
- To calculate the radiation, we must post-process the simulation
- Hot accretion flows are two-temperature (T_e, T_p are different) and probably not thermal → need
  - Prescription for heating of electrons vs protons/ions
  - Prescription for energy distribution/thermalization
- Major uncertainties, much work remains

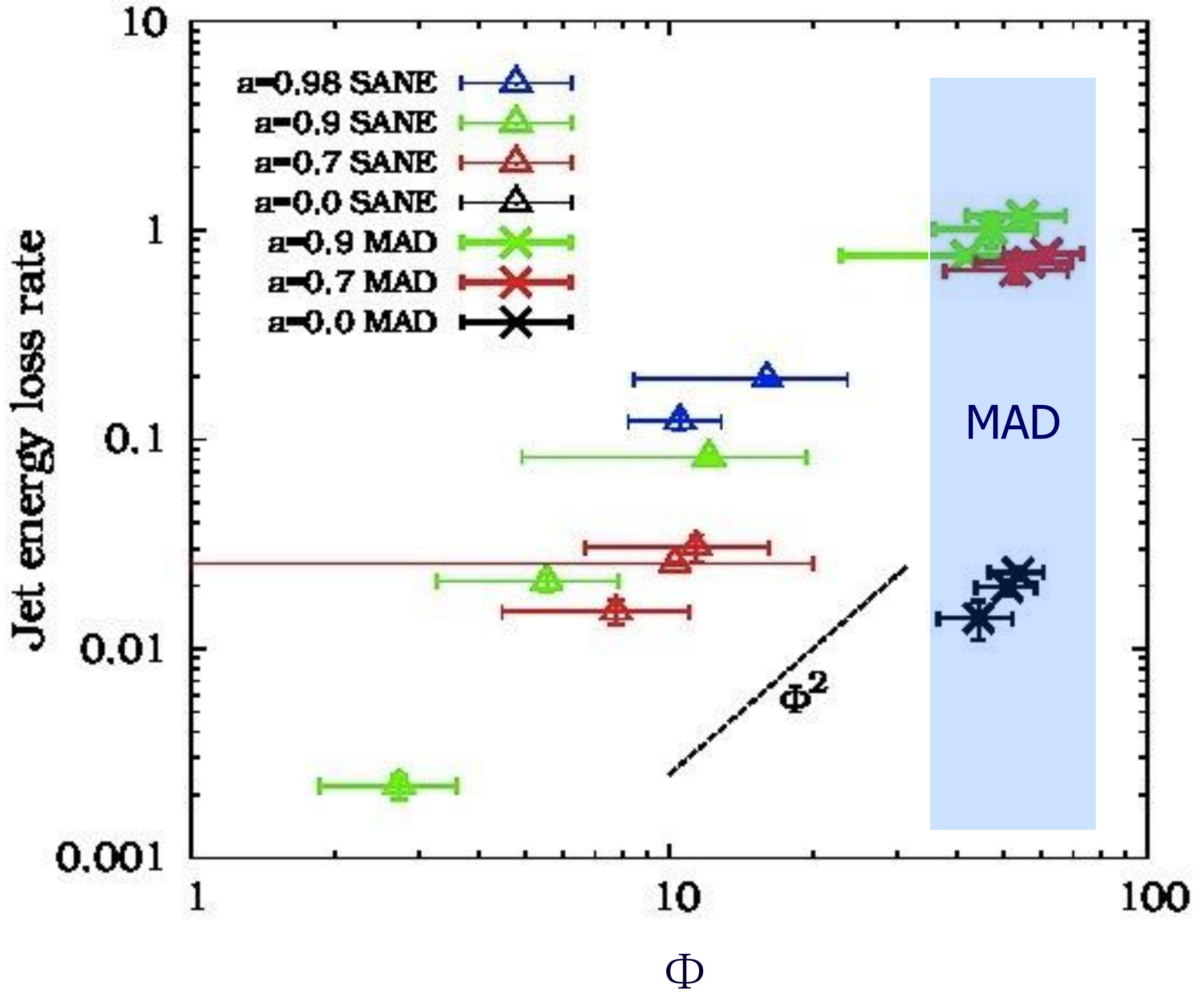
Jets: Theory and Numerical Simulations

- Jets form readily in simulations
- They are relativistic and powerful
- Jet power depends on BH rotation and magnetic flux at the horizon (Blandford & Znajek 1977; Ruffini & Wilson 1975)

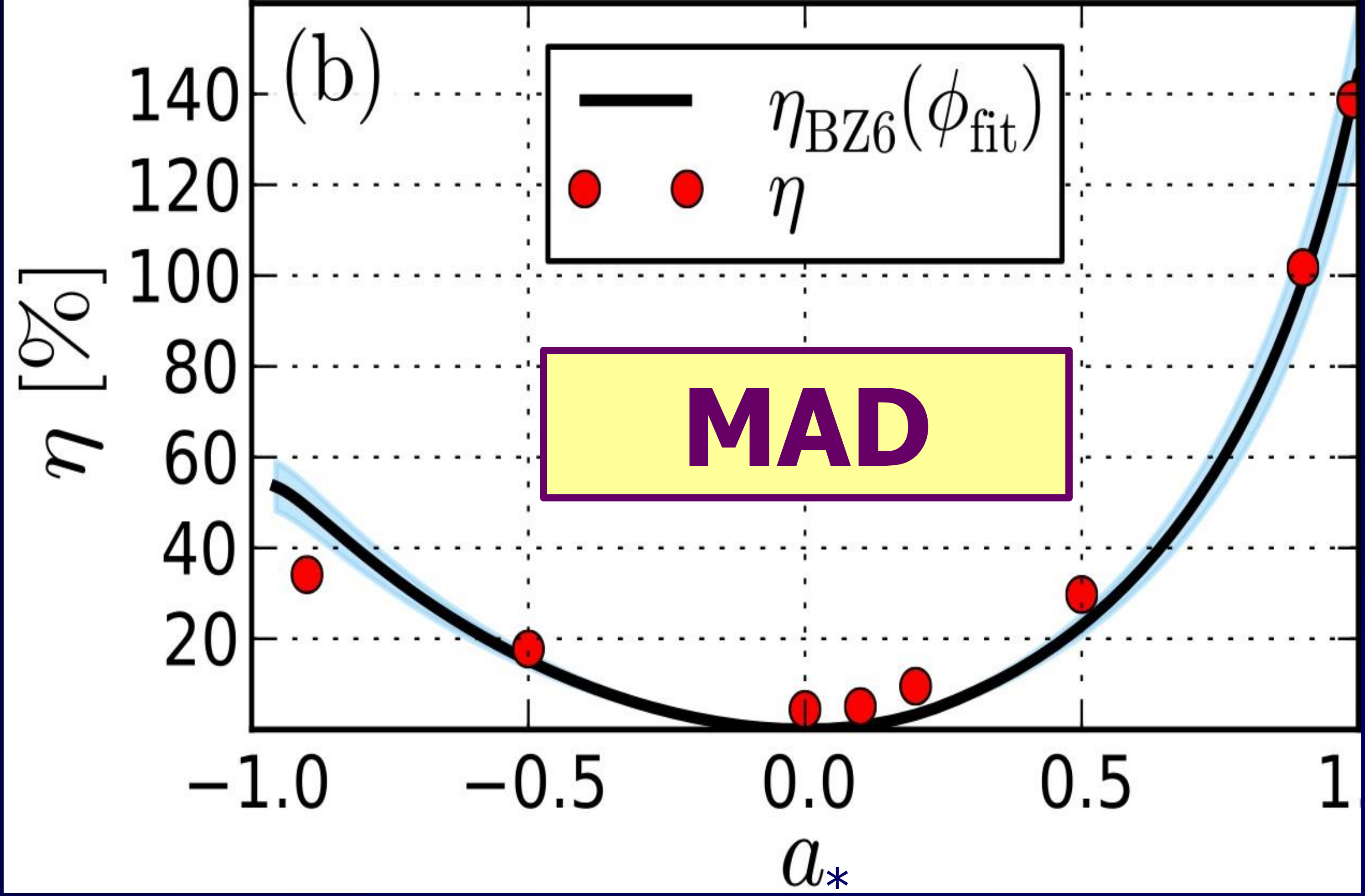
$$\boxed{P_{\rm jet} \approx \Phi_{\rm mag}^2 \Omega_{\rm H}^2 / c \propto \Phi_{\rm mag}^2 a_*^2}$$



3D GRMHD Simulation: Tchekhovskoy et al. (2011) **a_*=0.99**



Sądowski et al. (2014)



BH Jet in MAD (magnetically arrested disk) state can have a large efficiency: $\eta_{jet} = P_{jet}/\dot{M} c^2$ can even exceed 100% (Tchekhovskoy et al. 2011; 2012)

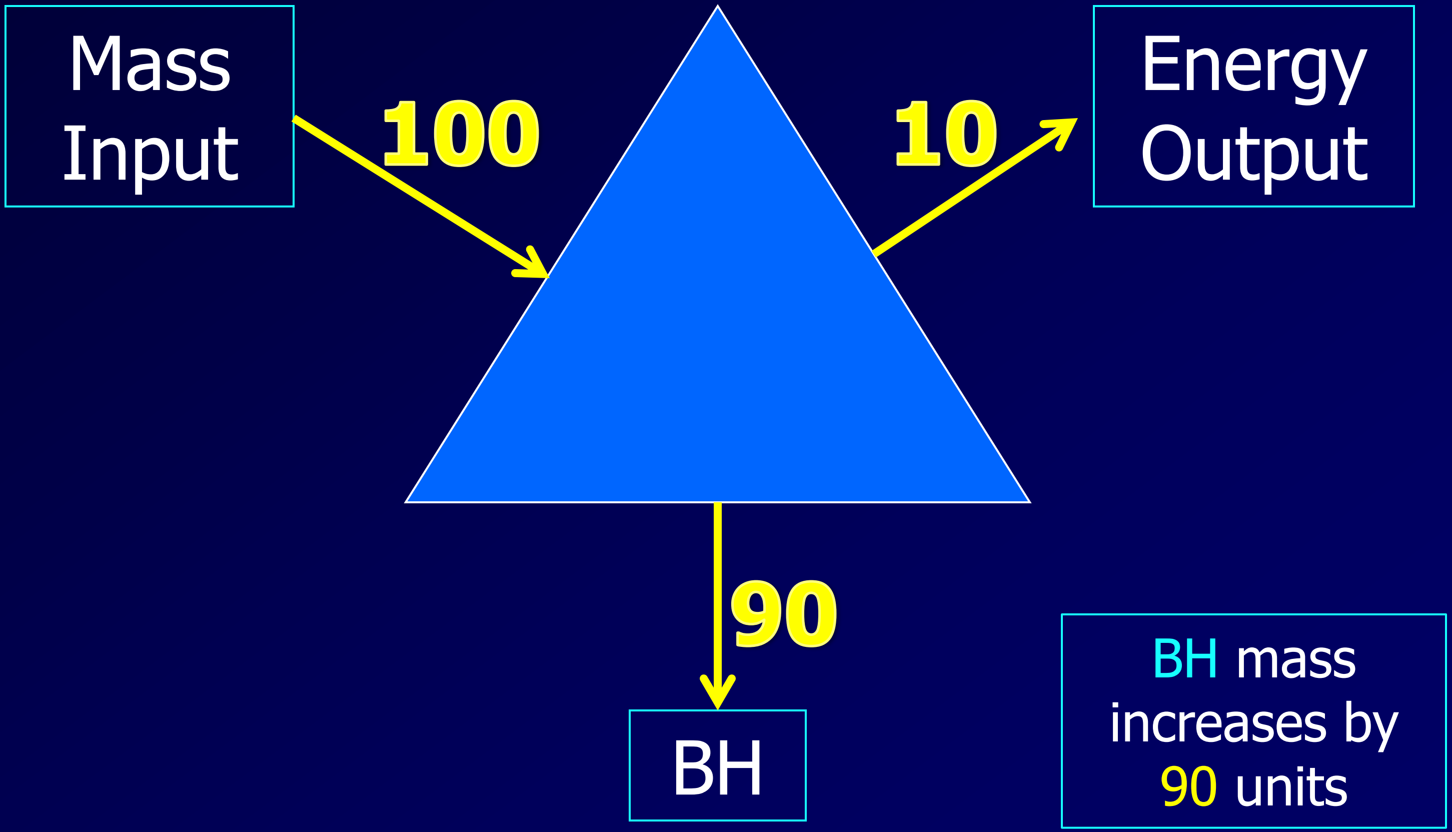
Strong dependence of η_{jet} on spin parameter a_*

Blandford-Znajek works beautifully on the computer

Is Jet Power from Accretion Disk or BH?

- This is a delicate question
- Gas falling into a potential well releases energy and can radiate: quasars
  - Simple physics
  - Nothing to do with BH energy extraction
  - Could jet be something similar?
  - Would involve no exotic physics…
- Or is jet powered directly by the BH?

Typical Accretion System



Mass Input

100

10

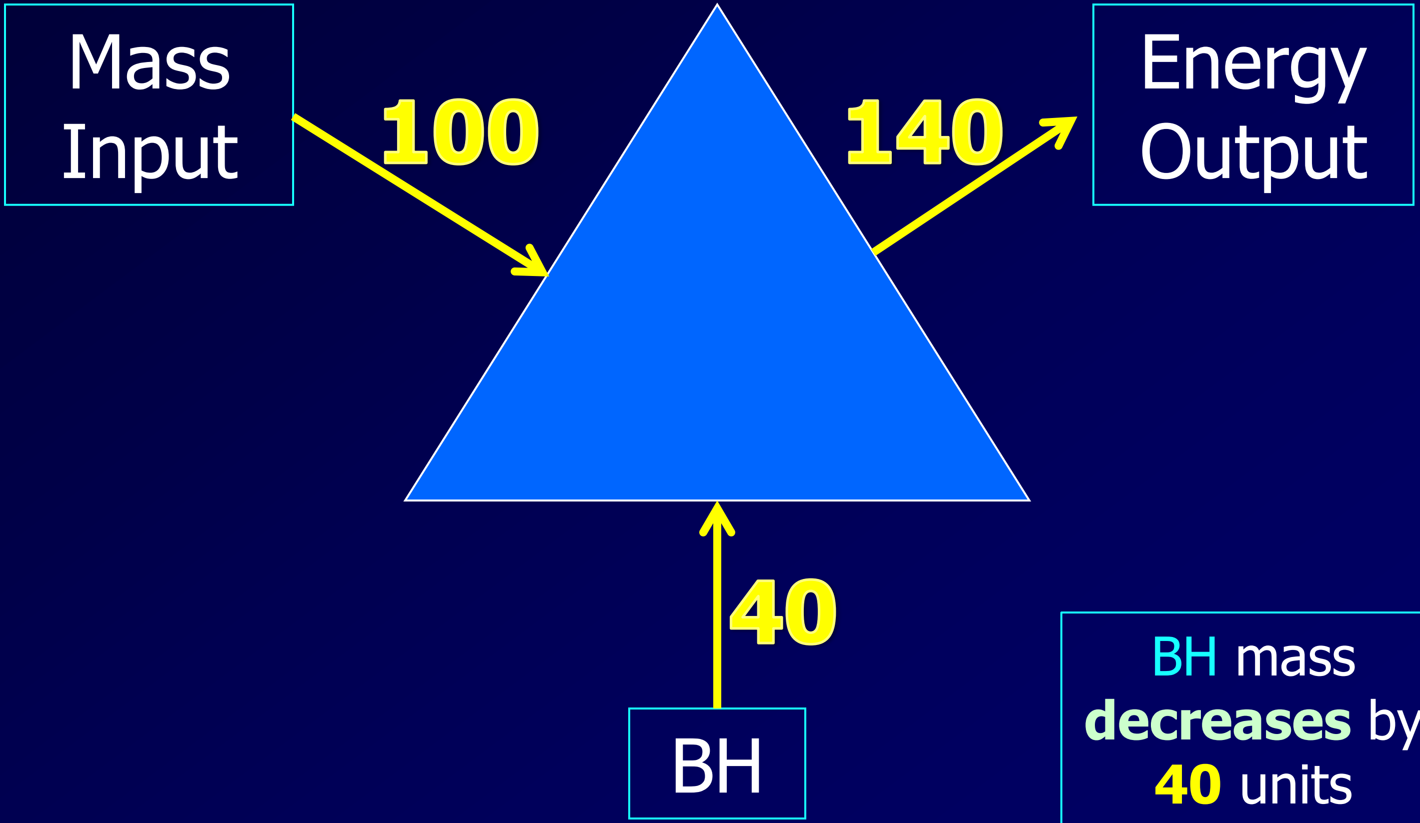
Energy Output

90

BH

BH mass increases by 90 units

Tchekhovskoy et al. (2011) Simulation



Mass Input

100

140

Energy Output

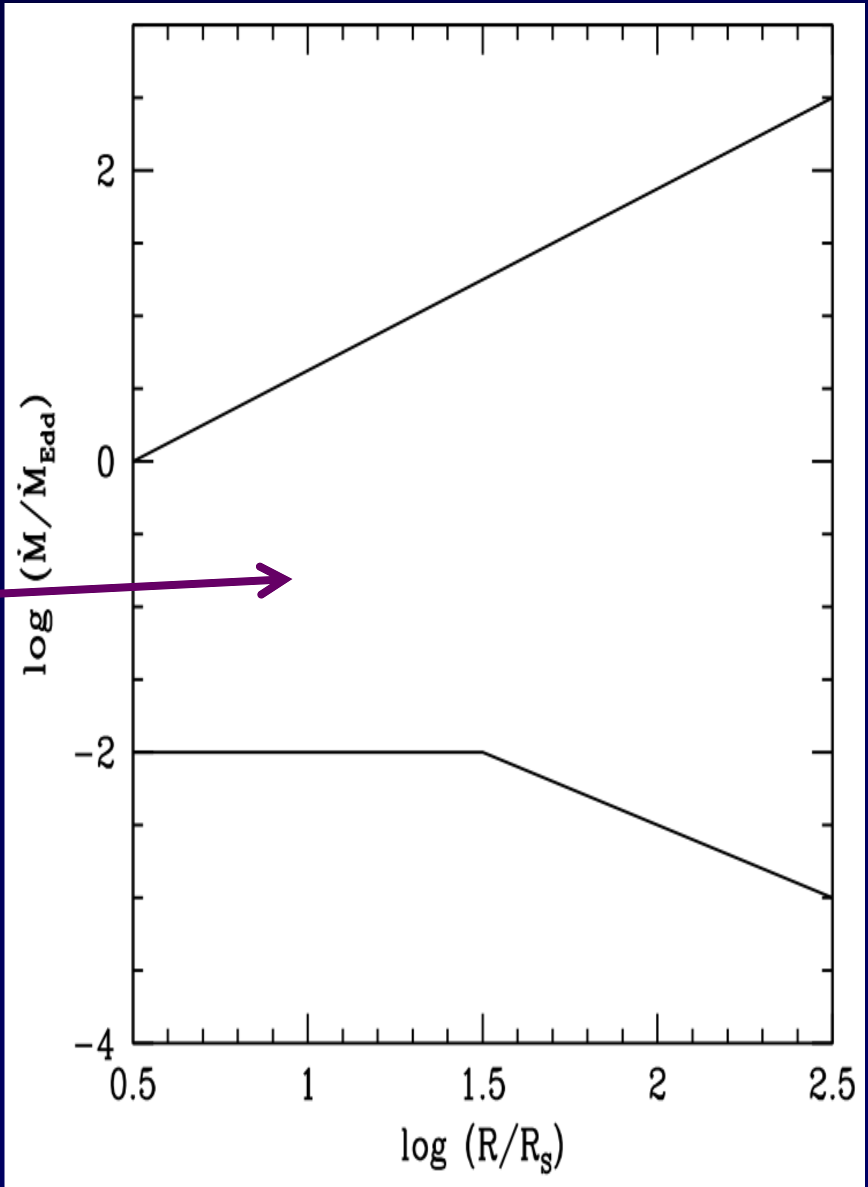
40

BH

BH mass **decreases** by **40** units

Accretion Regimes

Thin accretion disk: radiatively efficient
(Shakura-Sunyaev, Novikov-Thorne 73)
Typical QSOs, Seyferts
XRBs in thermal soft state

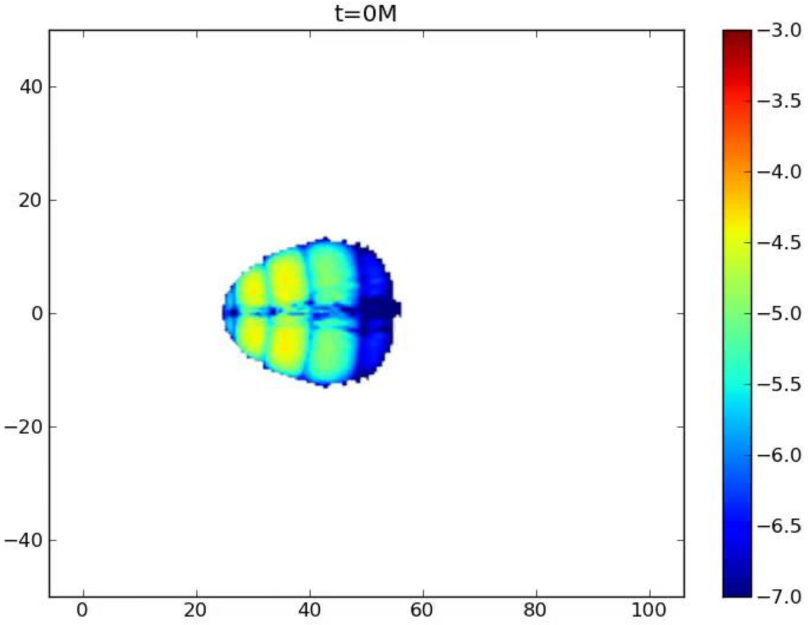


Thin Accretion Disk Model

Shakura & Sunyaev (1973)

Novikov & Thorne (1973)

- Self-consistent model that makes robust predictions for the radiative flux F(R) vs radius R
- Optically thick thermal gas, so relatively easy to compute the spectrum
- Excellent model for quantitative work, e.g., measuring BH spin (McClintock, Narayan,…)
- How good is the model really? (Krolik 1999)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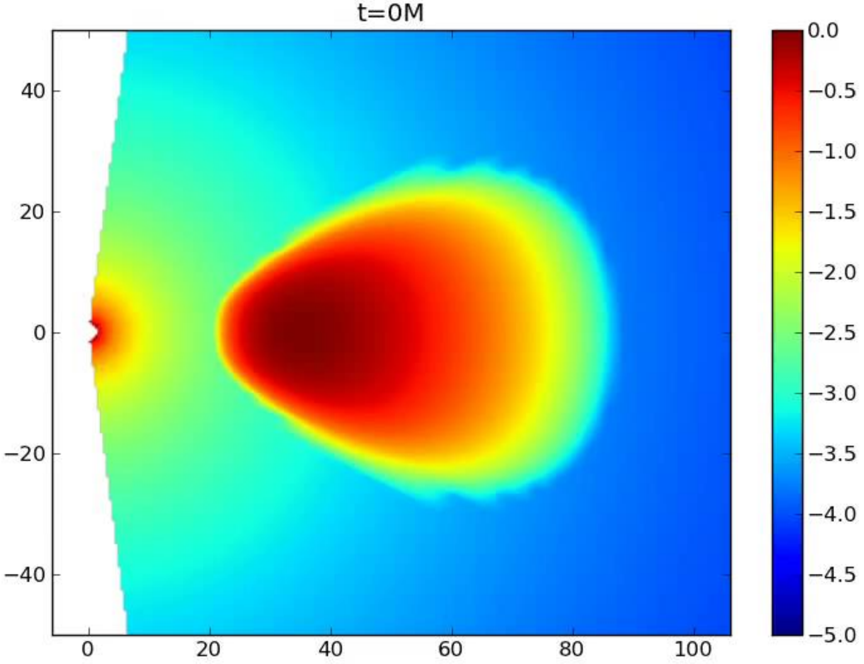
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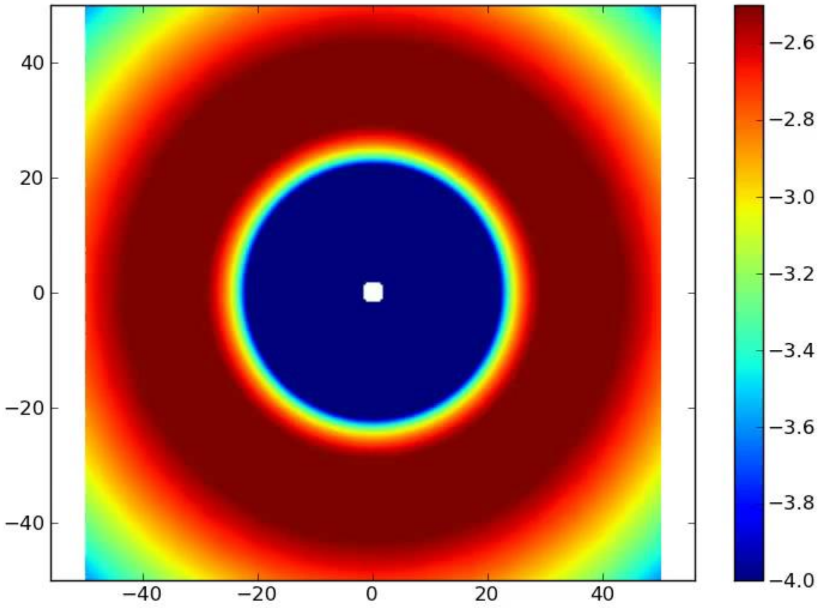
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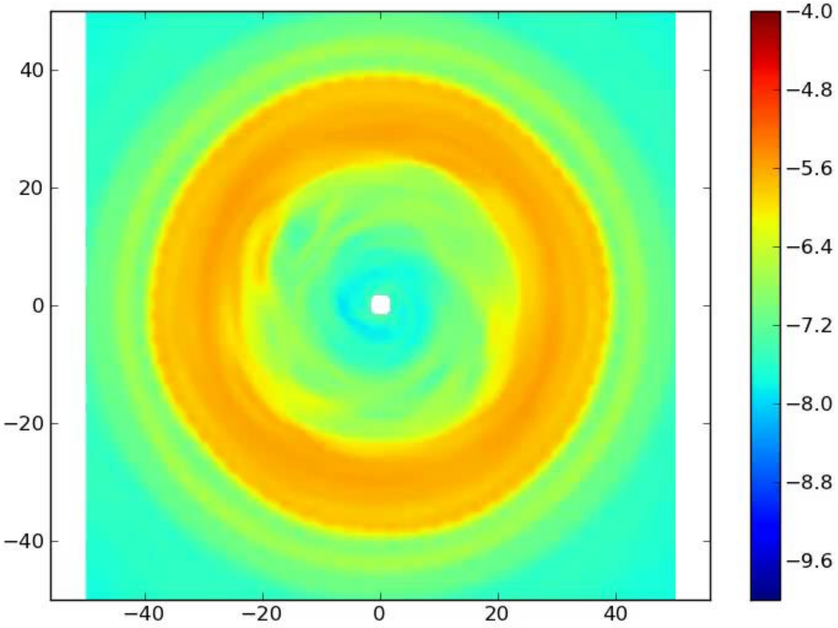
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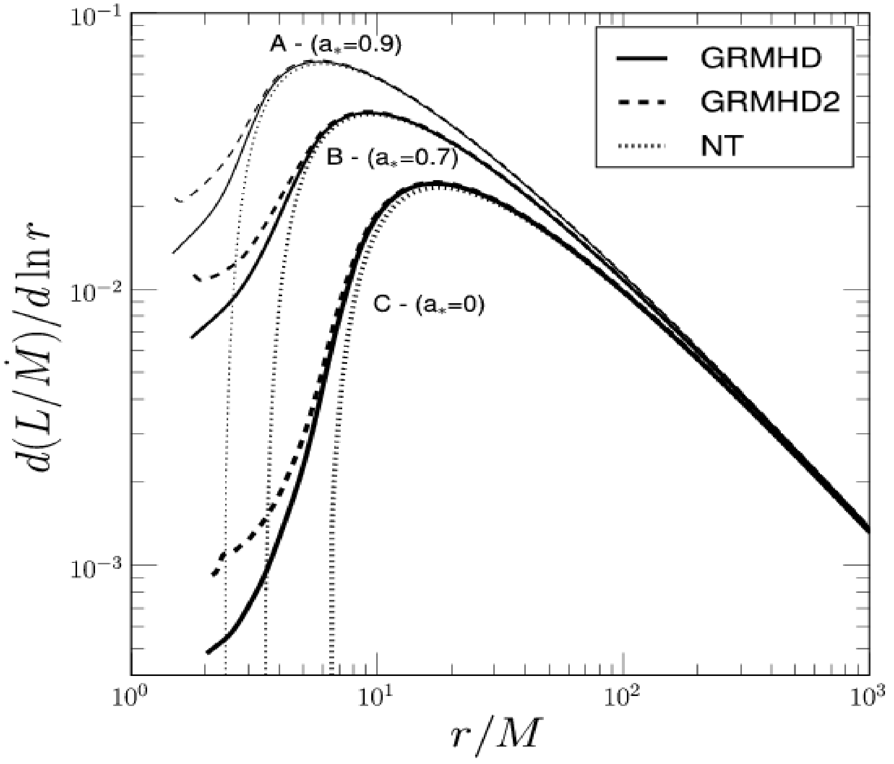
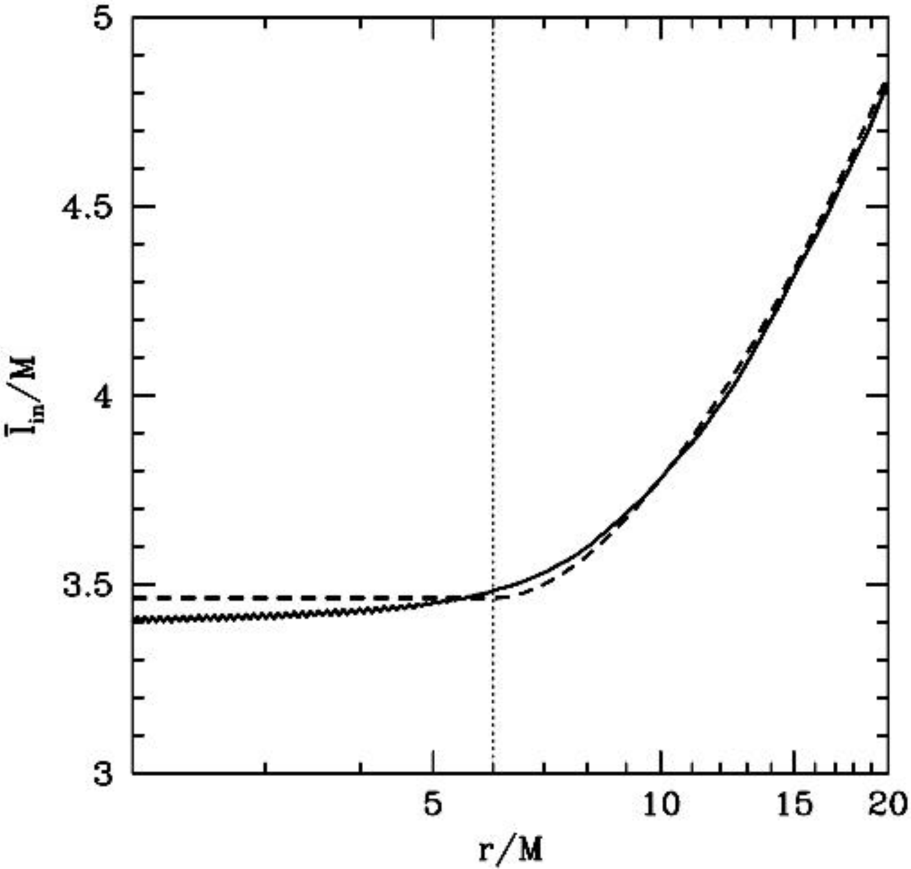




Shafee et al. (2008); Penna et al. (2010);
Kulkarni et al. (2011); Zhu et al. (2012)

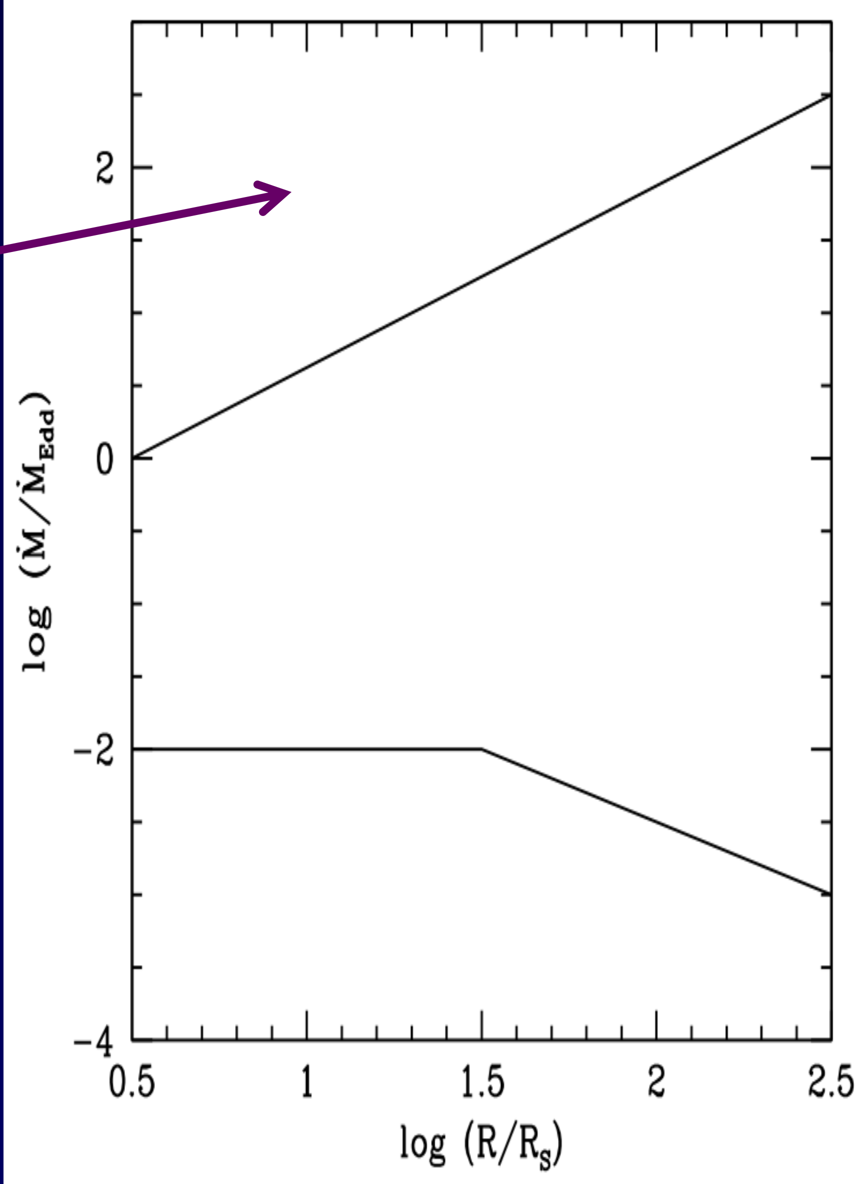
Thin disk simulations generally validate the analytical model of Novikov & Thorne (1973) for observables like luminosity, angular velocity

Are Thin Disks Stable?

- At luminosity > few percent Eddington, the thin disk model is radiation pressure dominated and thermally unstable (Shakura & Sunyaev 1976)
- Shearing sheet simulations by Hirose et al. (2009) suggested it is stable
- But improved radiation MHD simulations by Jiang et al. (2013) found instability
- Global disk models yet to be run…

Accretion Regimes

Hyper-accretion, slim disk, ADAF
(Abramowicz et al. 1989; N & Yi 94)
Super-Eddington accretion
TDEs, ULXs, SS433



Super-Eddington: Slim Disk: Hyper-Accretion Flow

- Mdot > Eddington
  - Radiation pressure is important
  - Optically very thick: $\tau >> 1$
  - Advection-dominated (ADAF/Slim Disk)
  - Puffed up: geometrically thick
- How common is it?
  - Probably common during early SMBH growth (e.g., Li 2012)
  - Also perhaps ULXs, TDEs, …

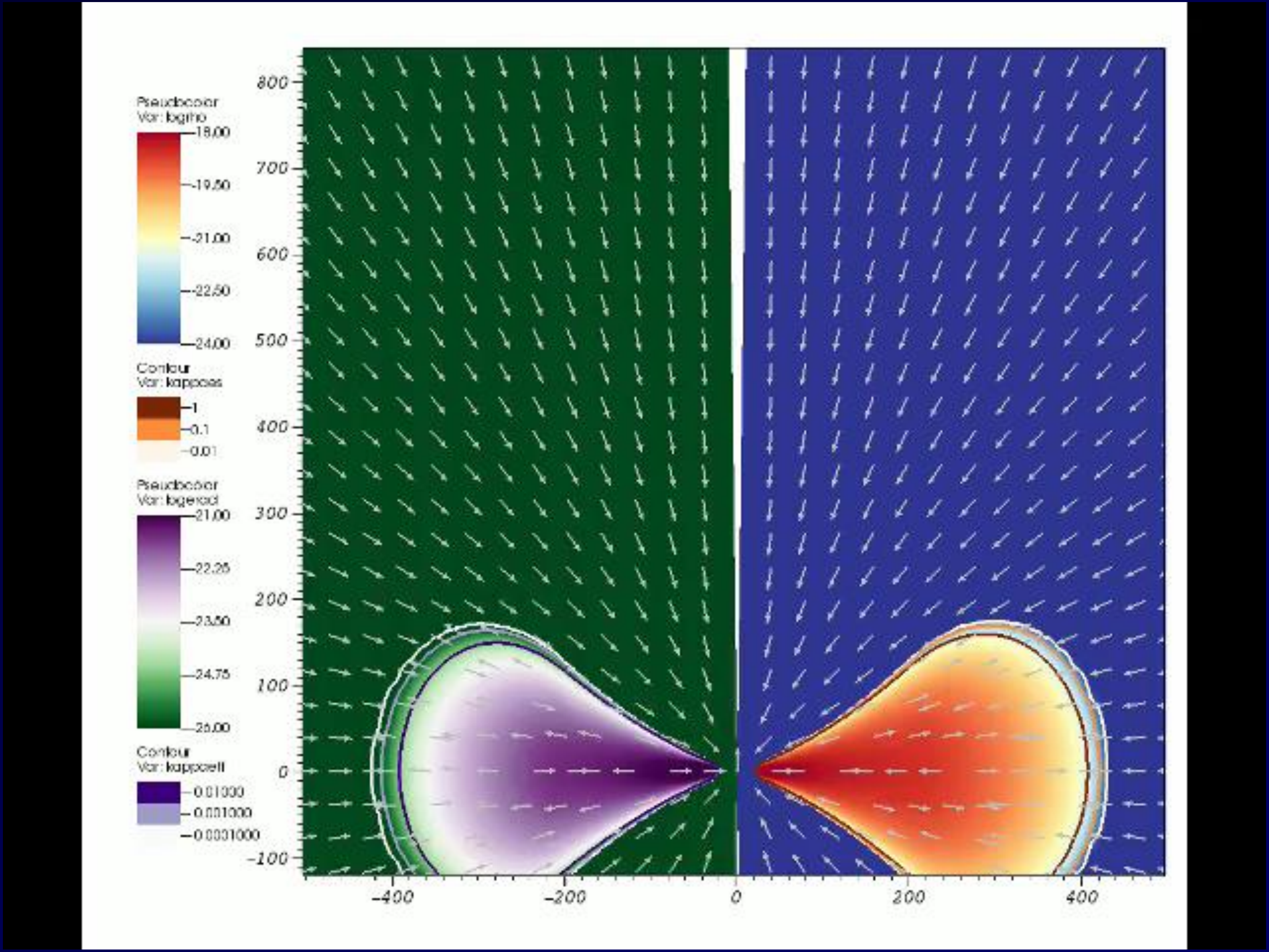
Long-Standing Questions

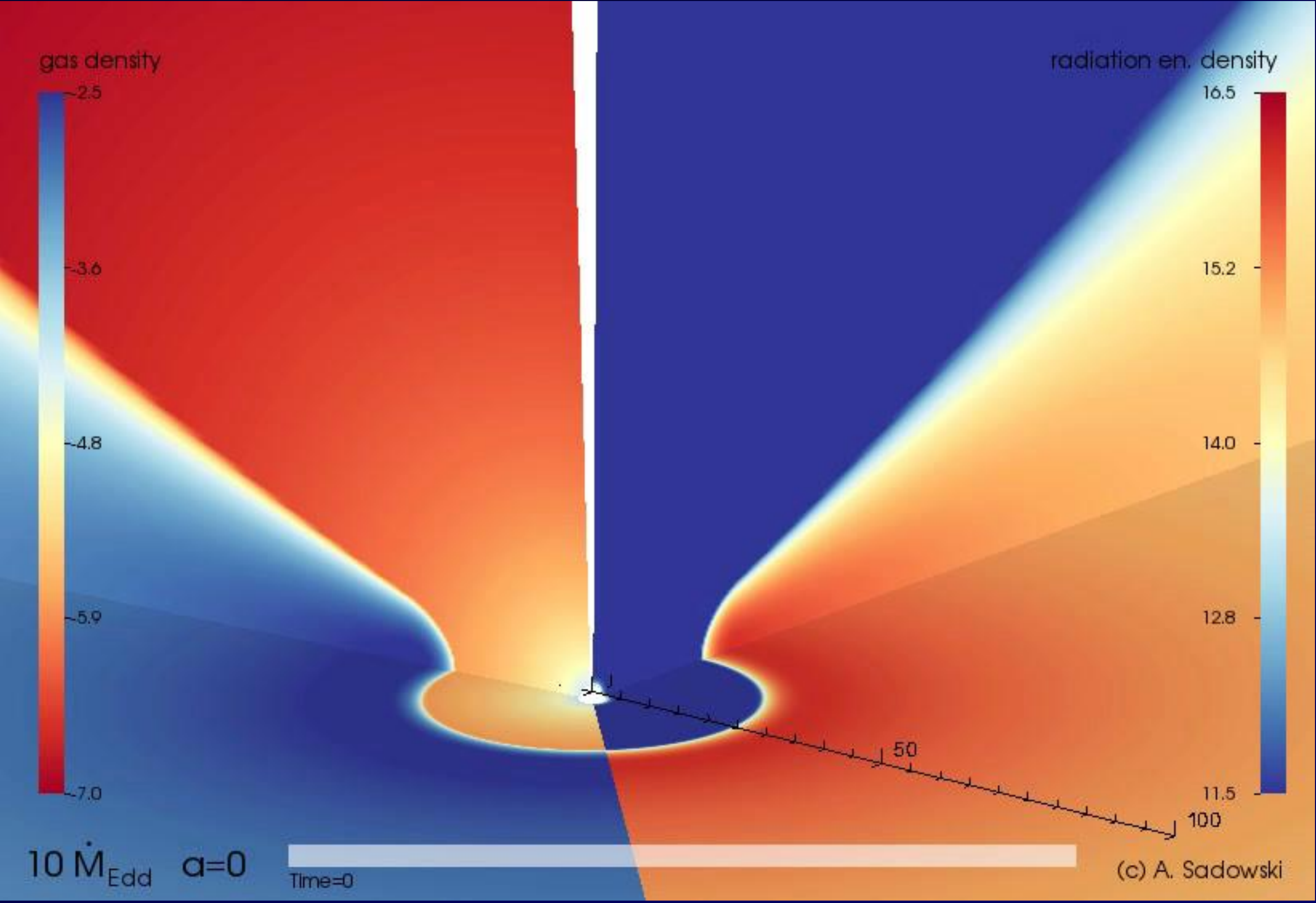
- Can a BH accrete at a rate above the Eddington mass accretion rate?
  - Crucial if we wish to understand how rapidly SMBHs grew in the early universe
- If super-Eddington accretion is possible, how luminous are these systems?
  - Limited to L_{Edd}?
  - If not, how bright can they be?
- Do they produce relativistic jets? Winds?
- Only simulations can provide answers

GRRMHD Simulations are Now Possible

- Achieved finally in 2014: Sądowski (KORAL) McKinney (HARMRAD) Fragile (Cosmos++)

GRRMHD simulation (2D): M=10$M_{\odot}$, a_{*}=0.9, Mdot=51Mdot$_{Edd}$, t ~100000M (Sądowski et al. 2014ab: KORAL)

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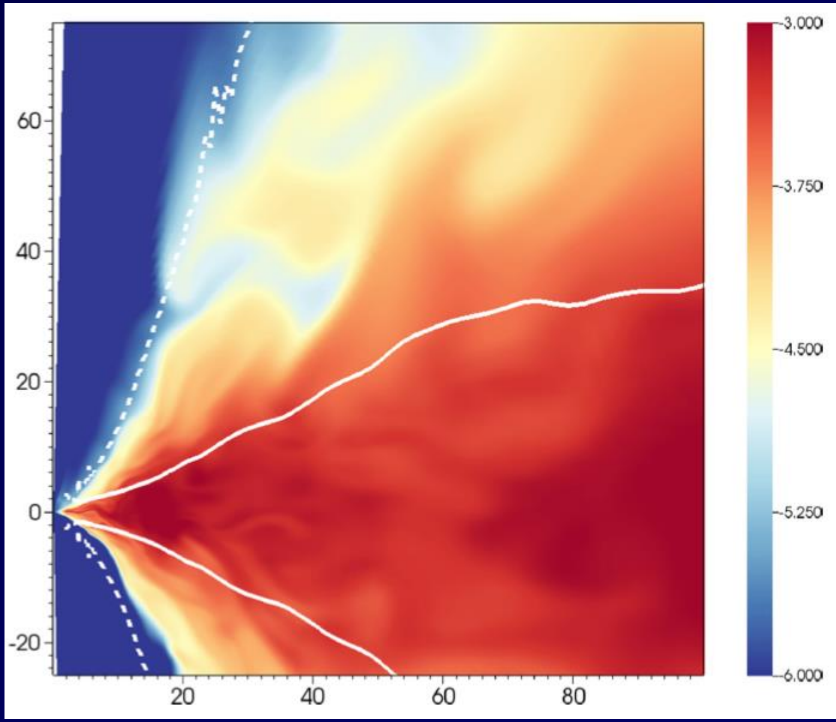
-3.6
-4.8
-5.9
-7.0
radiation en. density
16.5
15.2
14.0
12.8
11.5
50
100
10 $\dot{M}_{Edd}$ a=0
Time=0

(c) A. Sadowski

First Results

- Super-Edd accretion is possible
  - If you supply gas, the BH will eat it!
- Radiatively inefficient $L < \text{few } L_{Edd}$ (but Jiang et al. 2014 find more)
- Apparent luminosity looking down the funnel can be very large
  - Radiative Flux may be $10^3 F_{Edd}$
- A jet is always present
  - Even non-spinning BH has a jet
  - Blandford-Znajek process works, but it is not essential
- Super-Eddington accretion flows have Super-Eddington mechanical outflows (winds)



Sadowski et al. (2014ab, 2015)
McKinney et al. (2014)

Early Growth of SMBHs

- High z quasars with $M_{SMBH} \sim 10^{10} M_{\odot}$
- It is tough to make these SMBHs if accretion is Eddington-limited
- However, if there is enough gas supply(?), the SMBH can have Mdot >> Mdot_{Edd}
- Then no problem making massive BHs
- Expect powerful jets/winds: $P_{j,w} >> L_{Edd}$
- Observational consequences?
- Feedback and Mdot regulation?

Tidal Disruption Events

- According to standard models, TDEs have early super-Eddington accretion
- They should all have powerful jets
  - Reasonably broad beam
  - Modest Lorentz factor
- Current data are generally consistent…

Other Applications

- Ultra-Luminous X-ray Sources
  - Any evidence for or against jets?
- SS433, GRS1915+105
- Gamma-ray bursts
- …

Conclusion

- Black hole accretion disk simulations: Now a well-developed field
- Interesting results are coming out on disk structure and dynamics
- ISCO structure, jets, outflows, feedback
- Yet to be developed: Tools to calculate self-consistent disk spectra
  - Thermal spectra: near future
  - Non-thermal: not any time soon

Why is Radiation Hard?

- Radiation has to be handled as a separate fluid on top of magnetized gas
- Has its own speed, which can be very different
    - Serious problem because of Courant condition
    - Needs implicit techniques
- Have to deal with different opacity regimes
    - Optically thick: diffusion
    - Optically thin: free-streaming

Actually, GRRMHD is Not so Hard!

- In BH accretion, the magnetized gas is already relativistic
  - Radiation fluid has comparable velocity
  - Fully implicit techniques not needed --- semi-implicit is sufficient
- Relativistic four-notation and technology are actually a major help
  - Energy-momentum conservation is easy
  - No conceptual problems (as in Newtonian)

How Should We Represent the Radiation Field?

- At each instant, the radiation field is six-dimensional: $\mathbf{r}$, $\mathbf{n}$, ν
- Impractical to evolve the whole thing
- Simple prescriptions like diffusion or flux-limited diffusion are not good enough
- Simplest consistent scheme is M1: considers four bolometric quantities: U, **F**
- Straightforward closure: stress tensor $\mathbf{R}^{\mu\nu}$

Gas vs Radiation

- In deriving hydrodynamic equations, we consider ρ, $\rho\mathbf{v}$, etc., and close the equations with eqn of state: $p(\rho,T)$
- M1 is similar: U, $\mathbf{F}$ → $\mathbf{R}^{\mu\nu}$
- In hydrodynamics, viscosity has to be added separately via coefficients
- Same in radiation: can add radiation viscosity if needed